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SIMPLIFIED LONGITUDINAL-STABILITY ANALYSIS FOR A GLIDER IN TOWED FLIGHT

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More Important Symbols Used

$a = \frac{dC_z}{d\alpha}$ [1/rad] Variation in lift coefficient of glider as a function of the angle of incidence.

$a_1 = \frac{\partial C_{zH}}{\partial \alpha_H}$ [1/rad] Variation in lift coefficient of control surfaces as a function of the angle of incidence of the control surfaces.

$a_2 = \frac{\partial C_{zH}}{\partial \beta_H}$ [1/rad] Variation in lift coefficient of control surfaces as a function of angle of deflection of elevator.

C_{ma} Coefficient of moment inclining glider, deriving from aerodynamic forces.

C_{ml} Coefficient of moment inclining glider, deriving from towing cable.

C_n, C_t Aerodynamic coefficients of force normal and tangent to cable determined in relation to diameter and unit length of cable.

C_z Aerodynamic coefficient of aerodynamic lift.

C_x Aerodynamic coefficient of resistance.

d [m] Diameter of towing cable.

g m/sec^2 Acceleration of gravity.

h_z [m] Coordinate of towing attachment lock of glider measured vertically relative to center of gravity.

$\bar{h}_1 = -\frac{dC_{ma}}{dC_z}$ Static margin of glider in free flight.

$\bar{h}_k = -\frac{dC_m}{dC_z}$ Static margin of glider in towed flight.

$\bar{h}_l = -\frac{dC_{ml}}{dC_z}$ Variation in static margin of glider caused by towing.

see A67-22607 for original text

I_y [kGm sek^2] Moment of inertia of glider relative to transverse axis.

k_x [m] Coordinate of towing attachment lock of glider measured horizontally relative to center of gravity.

l_0, l_1 [m] Length of cable, free and under load.

l_a [m] Average aerodynamic chord.

l_H [m] Distance of axis of rotation of elevator from center of gravity of glider.

$m_u, m_w, m_{\dot{w}}, m_q$ Aerodynamic derivatives of moment of inclination relative to variation in longitudinal velocity, vertical velocity, rate of variation in angle of incidence, and angular velocity of inclination in dimensionless form.

m [kg sec^2/m] Mass of glider.

M [kgm] Moment inclining glider.

n [kg/m] Aerodynamic force normal to cable and acting on one meter of length.

P_n [kg] Aerodynamic force normal to cable.

P_t [kg] Aerodynamic force tangent to cable.

P_x [kg] Aerodynamic force of resistance.

P_z [kg] Aerodynamic buoyancy (lift).

P_{xH} [kg] Elevator drag.

P_{zH} [kg] Elevator lift.

q [kg/m] Unit weight of running meter of cable.

Q [kg] Glider weight.

$Re = dV/\nu$ Reynolds number.

S Lifting surface of glider wings.

S_H [m²] Lifting surface of elevator.

t [kg/m] Tangent aerodynamic force acting on one meter of cable length.

t [kg] Time.

T [kg] Cable tension.

T_1 [kg] Force deriving from cable and acting on glider towing attachment lock.

u [m/kg] Variation in horizontal velocity.

w [m/kg] Variation in vertical velocity.

$U_1 = V$ [m/kg] Flight speed.

x_s [m] Distance of center of gravity from aerodynamic center.

x_1 [m] Horizontal distance between ends of towing cable.

x_{x1}, x_{z1} Cable derivatives of horizontal component of force of tension of cable relative to horizontal and vertical displacement in dimensionless form.

x_u, x_w, x_q erodynamic derivatives of horizontal component of aerodynamic force relative to variations in horizontal, vertical, and angular velocity of inclination in dimensionless form.

X_1 [kg] Horizontal component of cable tension applied to glider towing attachment lock and acting in direction of flight

z_1 [m] Vertical distance between ends of towing cable.

γ Vertical distance of aerodynamic center from center of gravity.

Z_H [m] Vertical distance of axis of rotation of elevator from center of gravity.

Z_{x1}, Z_{z1} Cable derivatives of vertical component of force of cable tension relative to horizontal and vertical displacement in dimensionless form.

Z_u, Z_w, Z_q Aerodynamic derivatives of vertical component of aerodynamic force relative to variations in horizontal, vertical, and angular velocity of inclination in dimensionless form.

Z_1^l [kg] Vertical component of force of tension applied to glider towing attachment lock and acting normal to the direction of flight.

α [rad] Angle of incidence of glider.

α_{zH} [rad] Angle of incidence of vertical tail unit.

β_H [rad] Angle of elevator displacement.

$\varepsilon = 2C_z/\pi\Lambda_e$ [rad] Angle of displacement of streams flowing from wing.

φ_1 [rad] Angle of slope of towing cable relative to line of flight measured at the glider towing attachment lock.

φ_0 [rad] Angle of slope of towing cable relative to line of flight measured at aircraft towing attachment lock.

φ_P [rad] Towing angle limit due to equilibrium of forces.

φ_M [rad] Limit of towing angle due to equilibrium of moments.

λ [1/kg] Coefficient of elongation of cable.

Λ_e Aspect ratio of glider wing.

$\lambda = \xi \pm i\eta$ Roots of characteristic equation in dimensionless form.

$\varkappa_H = S_H l_H / S l_a$ Coefficient of effectiveness of elevator unit.

$\bar{\xi}$	Damping coefficient in dimensionless form.
$\bar{\eta}$	Oscillation frequency in dimensionless form.
ϱ [kg sec^2 m^{-4}]	Air density.

SIMPLIFIED LONGITUDINAL-STABILITY ANALYSIS FOR A GLIDER IN TOWED FLIGHT

Jerzy Maryniak

ABSTRACT. Discussion of the longitudinal stability of a rigid glider towed by an aircraft in straight horizontal steady flight. The configuration of the towing rope under the effect of aerodynamic forces is established, together with the equilibrium conditions of the glider. The range of angles formed by the towing rope and the corresponding lift coefficients are calculated. A stability analysis performed by a small perturbation technique proposed by Bryant, Brown, and Sweeting leads to a sixth-order characteristic equation, the roots of which may be determined by the Bairstow method. The results indicate that the towing process does not affect the high frequency oscillations but does enhance the fugoidal oscillations. To reduce the latter effect, it is essential to fly the glider below the level of the towing craft and to attach the towing rope in front and above the glider's center of gravity.

1. Introduction

Towed glider flights are widely employed at the present time. Up to the present, however, the problem of glider stability in towed flight has not been fully solved. There are only a few papers dealing with this problem.

In 1933 F. Janik [6] discussed the loads occurring in towed glider flight. This paper did not deal with stability problems. It determined the configuration of the towing cable, adopting approximate variation of the aerodynamic coefficients as a function of the angle of inclination of the cable, a variation not based on experimental results.

Reference [12] discussed the stability of a towed glider on a short "shaft" undergoing no deformation.

Reference [1] contains a number of publications of Bryant, Brown, and Sweeting relating to the stability of kites and gliders in towed flight. The authors in determining the towing angle (ϕ_1) dealt exclusively with the equilibrium of the forces, ignoring the equilibrium of the moments, this exerting a considerable influence on the value of the initial data necessary for dynamic stability calculations. While they introduced into the equations of motion the derivative of the pitching moment relative to variation in the vertical velocity

[1] Numbers in the margin indicate pagination in the foreign text.

$m_{\dot{\alpha}}$, as for free flight, they fail to allow for the variations deriving from towing. With some configurations these variations are considerable and affect the dynamic stability. In the paper in question the towing cable is treated as being inextensible.

S. Neumark in [9] discussed the configuration of an inextensible balloon cable on the basis of wind tunnel tests carried out in 1934 at the Aerodynamic Institute in Warsaw. In [10] dating from 1963 he discusses the problems of stability of a balloon on an inextensible cable. He derives the cable derivative, and determines the configurations of an inextensible cable, ignoring the resistance of friction. In the case of glider towing the sag of the towing cable is slight and the resistance of friction may be of the same order as the aerodynamic force normal to the cable and must not be ignored.

The present paper discusses the case of towing of a rigid (undeformable) glider by a heavier-than-air craft in level straight steady flight. On the basis of [9] and [10] the configuration is determined of an extensible towing cable, the resistance of friction being taken into account. The equilibrium and longitudinal static stability of a glider in towed flight are discussed, and the ranges of the towing angles and the coefficients of aerodynamic lift which can be achieved during towing are determined.

The theory of small perturbations has been applied in examination of the dynamic stability of a glider. Equations of motion have been obtained in the form of a system of ordinary second-order differential equations with constant coefficients [1]. This has permitted determination of the coefficients of the characteristic equation and application of the Routh-Hurwitz stability criteria, /58
as well as calculation of the roots of the characteristic equation by the Bairstow method.

The dynamics of the towing cable and the perturbations deriving from unstable movements of the aircraft have not been discussed. The problem has been solved by the method applied in dealing with stability in free flight [3, 4, 11]. This has permitted mutual comparison of the results corresponding to free flight and towed flight and has simplified analysis.

Stability analysis has been made on the basis of numerical calculations performed for a high-performance glider. The position relative to the towing aircraft, the towing speed, the location of the towing attachment lock relative to the center of gravity of the glider, and the length of the towing cable have been investigated from the standpoint of effect on glider stability.

2. Aerodynamics of Towing Cable

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In order to determine the configuration of the towing cable it is necessary to know the aerodynamic forces as well as to have data on the unit weight and the elastic properties of the cable. Because of the variability of the angle of slope of the cable in relation to the line of flight it is necessary to determine the coefficient of lift C_{z1} and the coefficient of resistance C_{x1} as a

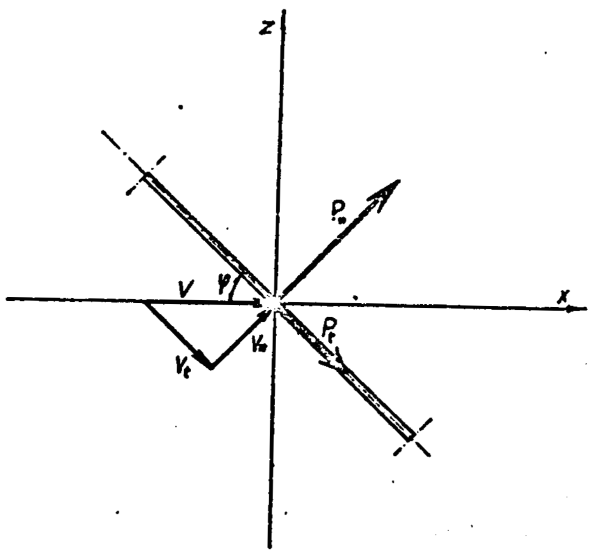

Figure 1. Distribution of Velocities and Aerodynamic Forces Acting on Cable Element of Length l and Diameter d.

function of the angle of slope of the cable ϕ.

The coefficient of aerodynamic resistance of a cable set perpendicular to the air flow (ϕ = 90°) is designated by C_n, and the coefficient of friction resistance of a cable set parallel in the flow (ϕ = 0), has been designated by C_t.

The aerodynamic forces acting on an element of cable of length l and diameter d inclined at an angle ϕ to the flow of a velocity V (Figure 1) has been considered.

The aerodynamic forces defined by the coefficients are of the form

$$P_n = \frac{1}{2}\varrho l d V^2 C_n \sin^2\varphi, \quad P_t = \frac{1}{2}\varrho l d V^2 C_t \cos^2\varphi.$$

resolving forces P_n and P_t in the direction normal and tangent to the flow we obtain:

$$P_{zl} = P_n\cos\varphi - P_t\sin\varphi = \frac{1}{2}\varrho l d V^2 (C_n\sin^2\varphi\cos\varphi - C_t\sin\varphi\cos^2\varphi),$$
$$P_{xl} = P_n\sin\varphi + P_t\cos\varphi = \frac{1}{2}\varrho l d V^2 (C_n\sin^3\varphi + C_t\cos^3\varphi). \qquad (2.1)$$

the lift acting on cable element P_{zl} and the force of resistance P_{xl} are determined by means of coefficients C_{zl} and C_x:

$$P_{zl} = \frac{1}{2}\varrho l d V^2 C_{zl}, \quad P_{xl} = \frac{1}{2}\varrho l d V^2 C_{xl}, \qquad (2.2)$$

where /61

$$C_{zl} = C_n \sin^2\varphi\cos\varphi - C_t \sin\varphi\cos^2\varphi,$$
$$C_{xl} = C_n \sin^3\varphi + C_t \cos^3\varphi. \tag{2.3}$$

The values of C_n and C_t have been selected so that relations (2.3) will agree with the experimental results of [9 and 13] (cf. Figure 2).

$$C_n = 1{,}15, \quad C_t = 0{,}035.$$

In Figure 2 the curves designated by N were obtained by S. Neumark during aerodynamic tests of balloon cables in 1934 at the Aerodynamic Institute in Warsaw [9]. On the other hand, the curves designated by the symbol W were given by K. D. Wood [13]. Functions (2.3) of the aerodynamic coefficients may be applied for towing cables over the entire range of velocities employed in towing gliders. The range of Reynolds numbers for towing cables embraces the values $Re = (1 \div 6)\cdot 10^4$. The maximum values of the Reynolds number are far below the critical Reynolds number $Re_{cr} = (1.8 \div 5)\cdot 10^5$ [2 and 13]. This gives us the assurance that we will always be in the subcritical range and that in calculations there is no need to conduct aerodynamic tests for individual towing cables [9].

3. Characteristics of Towing Cable

The following table gives the unit weights q of towing cables, the coefficients of elongation defined as $\lambda = \Delta l/Pl_0$, and the diameters d of cables employed at the present time in Poland for towing gliders [14].

Cable Type	Cable Description	d (m)	q (kg/m)	λ (1/kg)
A	Coiled stylon cable	0,0070	0,0308	0,00117
B	Braided stylon cable	0,0067	0,0326	0,00025
C	Braided stylon cable	0,0083	0,0462	0,00035
D	Coiled stylon cable	0,0113	0,0771	0,00021

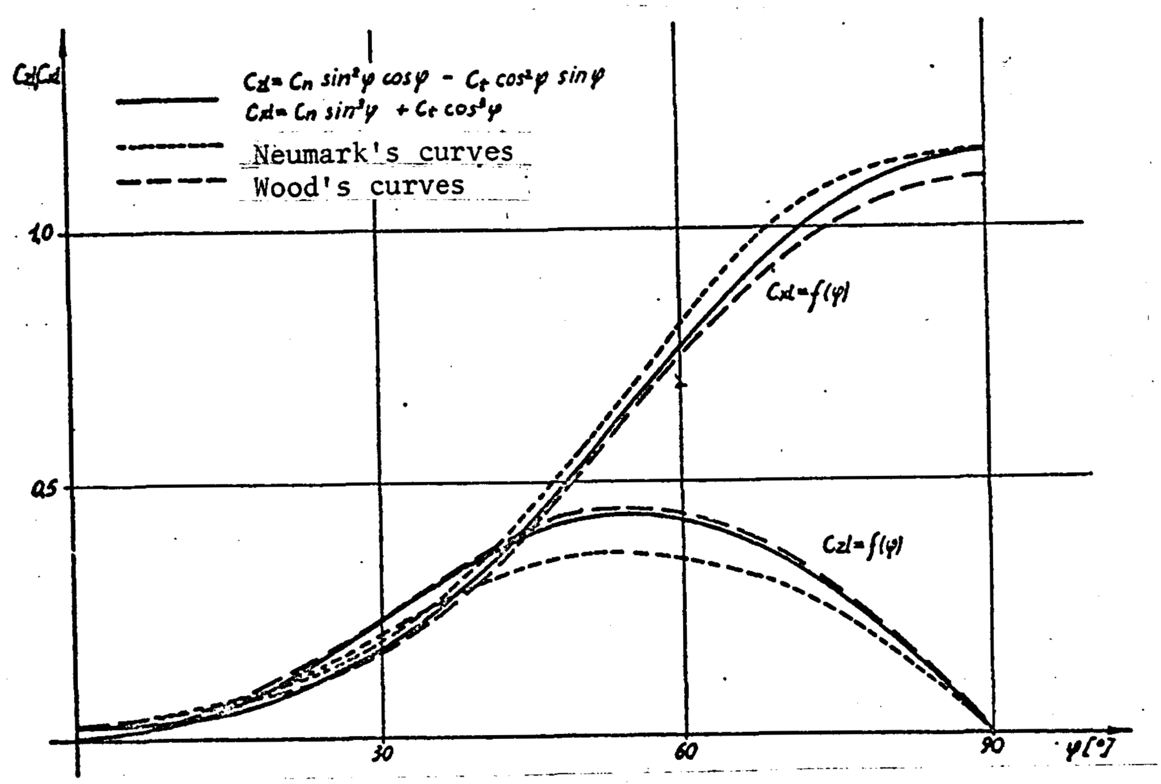

Figure 2. Experimental and Theoretical Curves of Aerodynamic Lift and Resistance Coefficients Versus Variation in Angle of Inclination

4. Configuration of Towing Cable

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In considering the problems of equilibrium and stability of gliders in towed flight it is necessary to know the forces T_1 on the glider towing lock attachment as a function of the coordinates of the ends of the towing cable, x_1, z_1, and angles ϕ_0 and ϕ_1 relative to the line of flight, which the cable creates on the towing attachments of the aircraft and the glider (Figure 3).

Let us consider the case of towing of a glider in which the towing aircraft is in steady, horizontal, straight flight, while the glider may assume any position in the vertical plane in the line of flight.

The towing cable is regarded as an ideally flexible and heavy connector to which aerodynamic forces are applied. The bending moments due to rigidity of the cable may be ignored in the case of a towing cable [5, 7, 8].

A cable element dl has been considered to which is applied the force of tension of the cable T and T + dT, dead weight qdl, tangent aerodynamic force

$t \cos^2 \phi dl$, and normal aerodynamic force $n \sin^2 \phi dl$ have been applied (Figure 4). The following notation has been introduced:

$$n = \frac{1}{2}\varrho dV^2 C_n, \quad t = \frac{1}{2}\varrho dV^2 C_t.$$

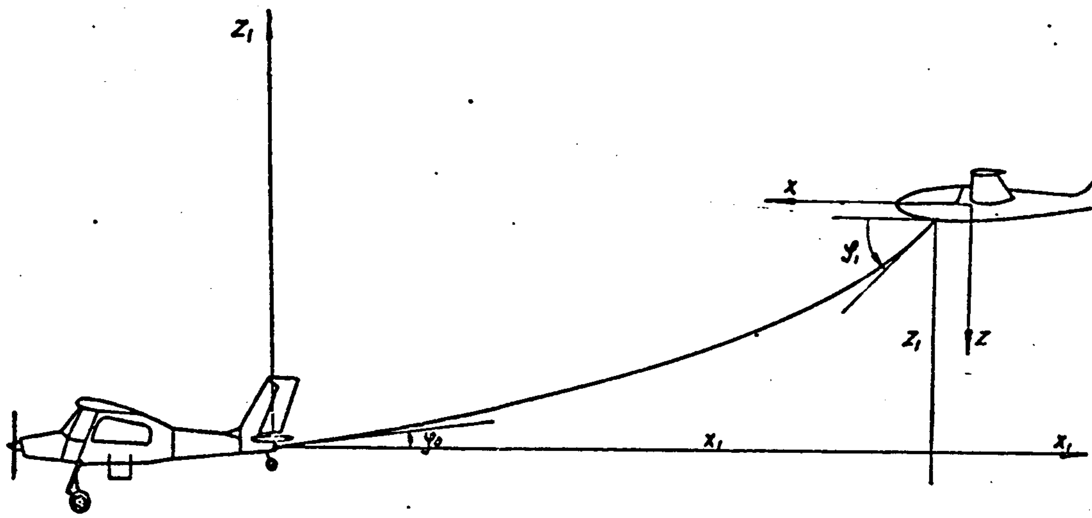

Figure 3. Geometric Quantities Characterizing the Towing of a Glider

The equations for projection of the forces in the directions tangent and normal to the cable element are of the form

$$\begin{aligned} &-T+(T+dT)\cos(d\varphi)+t\cos^2\varphi\, dl-q\sin\varphi\, dl=0, \\ &-(T+dT)d\varphi+n\sin^2\varphi\, dl+q\cos\varphi\, dl=0. \end{aligned} \qquad (4.1)$$

After conversions, the small terms of a higher order being ignored, and introduction of the dimensionless coefficient of cable weight /63

$$C_q = \frac{q}{\frac{1}{2}\varrho dV^2}$$

we obtain

$$\frac{dT}{T}=\frac{\frac{C_q}{C_n}\sin\varphi-\frac{C_t}{C_n}\cos^2\varphi}{\frac{C_q}{C_n}\cos\varphi+\sin^2\varphi}\,d\varphi. \tag{4.2}$$

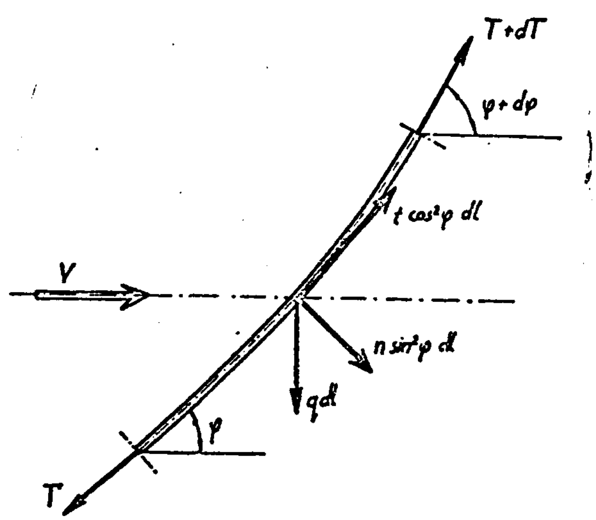

Figure 4. Distribution of Forces Acting on Cable Element of Length dl

After introducing the new constants [10]

$$\frac{C_q}{C_n}=2\operatorname{ctg}2\psi,\quad \frac{C_t}{C_n}=2\frac{C_t}{C_q}\operatorname{ctg}2\psi$$

and integrating (4.2) we obtain the relation

$$\frac{Te^{\eta}}{\tau}=\frac{T_1e^{\eta_1}}{\tau_1}=K_1, \tag{4.3}$$

where

$$\tau=\left(\frac{\operatorname{ctg}\psi-\cos\varphi}{\operatorname{tg}\psi+\cos\varphi}\right)^{\cos 2\psi} \tag{4.4}$$

$$\eta=\frac{C_t}{C_q}\cos 2\psi\left[-\frac{2\varphi}{\sin 2\psi}+\frac{\operatorname{tg}^2\psi}{\sqrt{1-\operatorname{tg}^2\psi}}\operatorname{arctgh}\left(\sqrt{\frac{1-\operatorname{tg}\psi}{1+\operatorname{tg}\psi}}\operatorname{tg}\frac{\varphi}{2}\right)+\right.$$
$$\left.+\frac{2\operatorname{ctg}^2\psi}{\sqrt{\operatorname{ctg}^2\psi-1}}\operatorname{arctg}\left(\sqrt{\frac{\operatorname{ctg}\psi-1}{\operatorname{ctg}\psi+1}}\operatorname{tg}\frac{\varphi}{2}\right)\right]. \tag{4.5}$$

As with τ and η we obtain the functions of τ_1 and η_1, substituting ϕ_1 in place of ϕ in (4.4) and (4.5). /64

By applying relation (4.3) we can calculate the cable tension at any point of the cable, provided we know the value of ϕ and T at one of its ends.

From the second equation of (4.1) we obtain

$$Td\varphi = (q\cos\varphi + n\sin^2\varphi)dl. \tag{4.6}$$

after substituting relation (4.3) in (4.6) we obtain the length of the towing cable in the integral form

$$l_1 = \frac{T_1 e^{\eta_1}}{\tau_1} \frac{2}{\varrho d V^2} \int_{\varphi_0}^{\varphi_1} \frac{\tau e^{-\eta}}{C_q\cos\varphi + C_n\sin^2\varphi} d\varphi. \tag{4.7}$$

The coordinates of the ends of the towing cable x_1 and z_1 (Figure 3) are calculated on the basis of the geometric relations

$$dx_1 = dl\cos\varphi, \quad dz_1 = dl\sin\varphi;$$

we obtain

$$x_1 = \frac{T_1 e^{\eta_1}}{\tau_1} \frac{2}{\varrho d V^2} \int_{\varphi_0}^{\varphi_1} \frac{\tau e^{-\eta}\cos\varphi}{C_q\cos\varphi + C_n\sin^2\varphi} d\varphi, \tag{4.8}$$

$$z_1 = \frac{T_1 e^{\eta_1}}{\tau_1} \frac{2}{\varrho d V^2} \int_{\varphi_0}^{\varphi_1} \frac{\tau e^{-\eta}\sin\varphi}{C_q\cos\varphi + C_n\sin^2\varphi} d\varphi. \tag{4.9}$$

We change the limits of integration in order to introduce the new functions

$$\mu = \frac{2}{\varrho d V^2} \int_0^{\varphi} \frac{\tau e^{-\eta}}{C_q\cos\varphi + C_n\sin^2\varphi} d\varphi, \tag{4.10}$$

$$\sigma = \frac{2}{\varrho d V^2} \int_0^{\varphi} \frac{\tau e^{-\eta}\cos\varphi}{C_q\cos\varphi + C_n\sin^2\varphi} d\varphi, \tag{4.11}$$

$$\nu = \frac{2}{\varrho d V^2} \int_0^\varphi \frac{\tau e^{-\eta} \sin\varphi}{C_q \cos\varphi + C_n \sin^2\varphi} d\varphi. \tag{4.12}$$

The length of the towing cable is determined by use of functions (4.10) to (4.12) and relation (4.3):

$$l_1 = K_1(\mu_1 - \mu_0) \tag{4.13}$$

and the coordinates of the end of the cable at the glider towing attachment relative to the towing attachment of the towing aircraft:

$$x_1 = K_1(\sigma_1 - \sigma_0), \tag{4.14}$$

$$z_1 = K_1(\nu - \nu_0). \tag{4.15}$$

for the extensible cables employed for towing the following has been adopted:

$$l_1 = l_0(1 + \lambda T_1); \tag{4.16}$$

by taking into account the changes in tension T along the cable we obtain the differences of ≈ 0.01% in relation to l_1 calculated from (4.16) and ≈ 3% in relation to Δl_1 (elongation).

Figures 5a and 5b give examples of the functions μ, σ, ν, τ, and e^{η} obtained after numerical integration of (4.10) to (4.12) for a type C cable and a towing speed V = 20 m/sec and 50 m/sec. /67

The towing angle ϕ_1 and the C_z of towing corresponding to it we determine on the basis of section 6, this permitting calculation of the force on the glider towing attachment T_1. From relation (4.16) we calculate the total length of the cable l_1, and from the diagrams of Figure 5 for the value of ϕ_1 we find μ_1, σ_1, and ν_1, this permitting calculation of μ_0 from equations (4.3) and (4.13). From the diagram in Figure 5 we find for the value μ_0 the angle ϕ_0, and then σ_0 and ν_0. Using (4.14) and (4.15) we calculate the coordinates of the ends of the cable x_1 and z_1.

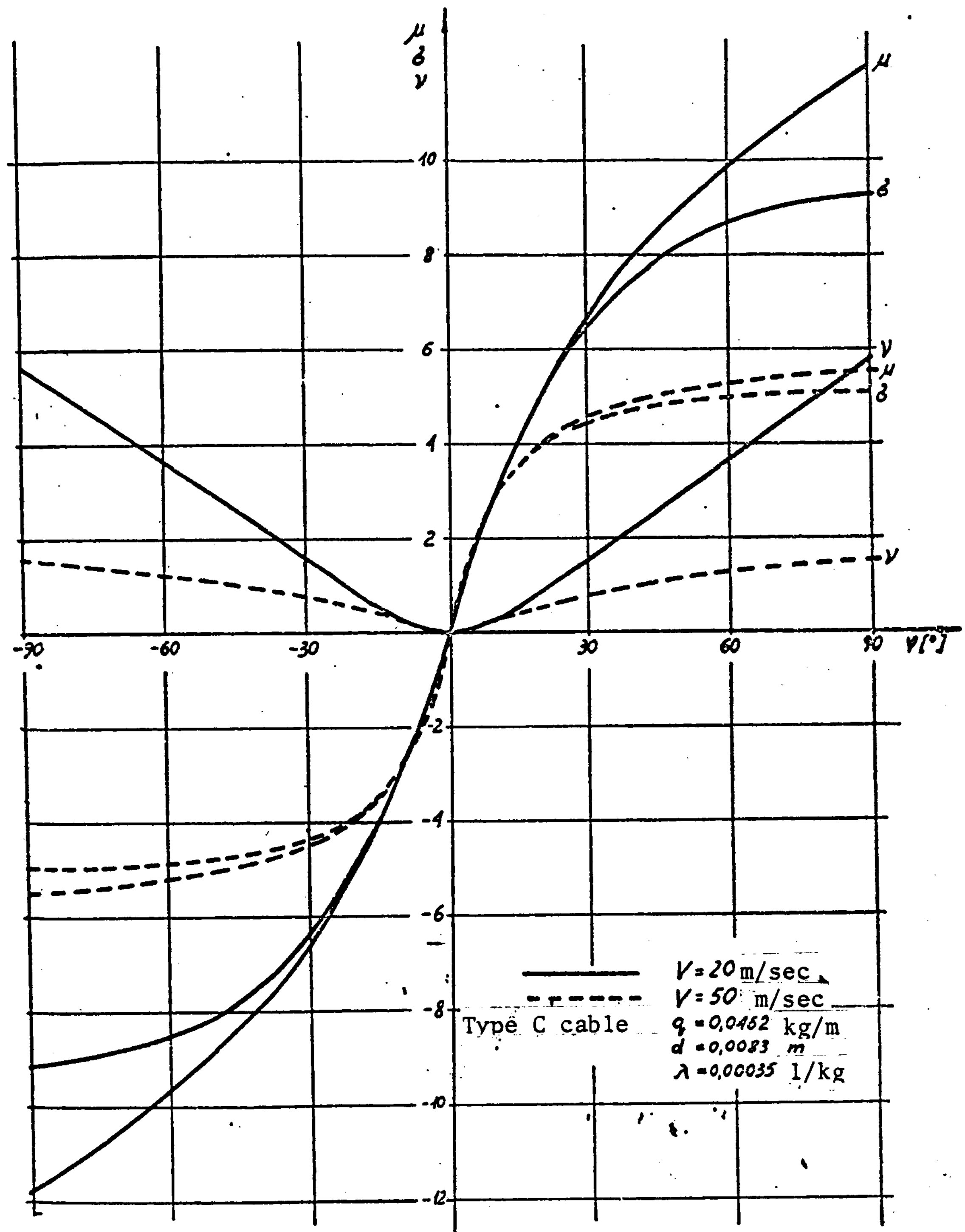

Figure 5a. Functions of Length μ and Coordinates of Ends of Cable σ and ν for Type C Cables at Towing Speeds of 20 and 50 m/sec

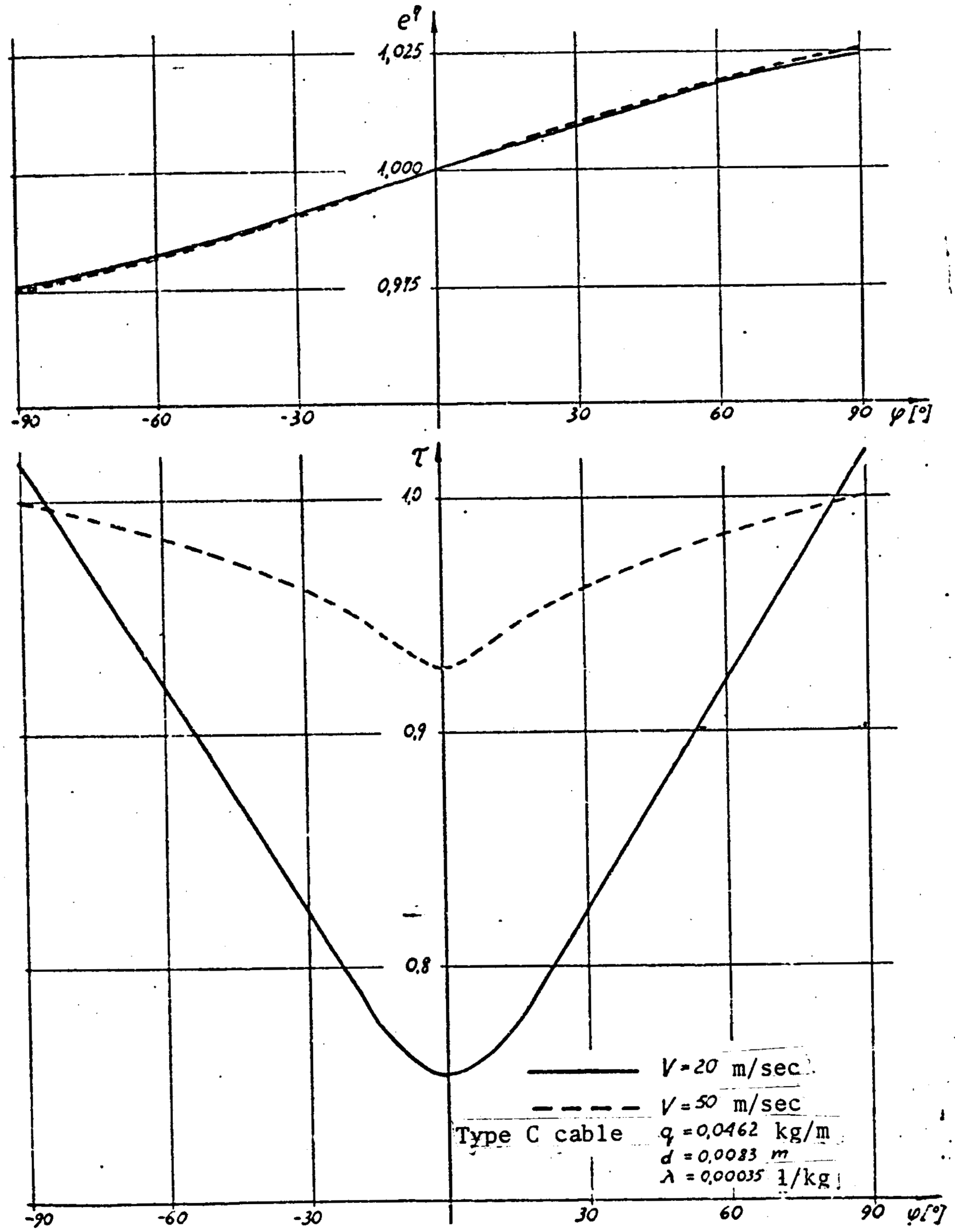

Figure 5b. Functions τ and e^{η} for Type C Cable at Towing Speeds of 20 and 50 m/sec

5. Coefficients of Forces Deriving from Towing Cable (Cable Derivatives) /67

In order to determine the forces deriving from the towing cable we assume the variation in the forces with the displacement of the ends of the cable relative to each other to be linear in nature.

On the analogy of the aerodynamic derivatives employed in investigation of the stability of aircraft, we have introduced the cable derivatives (coefficients of forces) which are defined as follows:

$$X^l_{x_1} = \frac{\partial X^l_1}{\partial x_1}, \quad X^l_{z_1} = \frac{\partial X^l_1}{\partial z_1},$$
$$Z^l_{x_1} = \frac{\partial Z^l_1}{\partial x_1}, \quad Z^l_{z_1} = \frac{\partial Z^l_1}{\partial z_1}.$$

The variations in the forces deriving from the towing cable are represented by means of the cable derivatives

$$\begin{aligned} dX^l_1 &= X^l_{x_1} dx_1 + X^l_{z_1} dz_1, \\ dZ^l_1 &= Z^l_{x_1} dx_1 + Z^l_{z_1} dz_1. \end{aligned} \tag{5.1}$$

Force T_1 deriving from the towing cable is applied to the attachment at angle ϕ_1 relative to the line of flight, and its corresponding components are:

$$X^l_1 = T_1 \cos\varphi_1, \quad Z^l_1 = T_1 \sin\varphi_1. \tag{5.2}$$

We obtain the variation in the force after differentiation of relation (5.2):

$$\begin{aligned} dX^l_1 &= dT_1 \cos\varphi_1 - T_1 d\varphi_1 \sin\varphi_1, \\ dZ^l_1 &= dT_1 \sin\varphi_1 + T_1 d\varphi_1 \cos\varphi_1. \end{aligned} \tag{5.3}$$

If we introduce into (5.3) the dependence on dT_1 and $T_1 d\phi_1$ expressed by the elementary displacements dx_1 and dz_1 and T_1, l_1, x_1, z_1, ϕ_1, and ϕ_0, after equation with (5.1) we calculate the cable derivatives $X^l_{x_1}$, $X^l_{z_1}$, $Z^l_{x_1}$, and $Z^l_{x_1}$.

After converting (4.2) and (4.3) and introducing them into (4.6), we obtain the dependence on the elementary cable length

$$dl = \frac{T_1}{\tau_1} e^{\eta_1-\eta} \frac{\tau\tau'}{C_q \sin\varphi} \frac{2}{\varrho d V^2} d\varphi, \tag{5.4}$$

and similarly

$$dx = \frac{T_1}{\tau_1} e^{\eta_1-\eta} \frac{\tau\tau' \cos\varphi}{C_q \sin\varphi} \frac{2}{\varrho d V^2} d\varphi, \tag{5.5}$$

$$dz = \frac{T_1}{\tau_1} e^{\eta_1-\eta} \frac{\tau\tau'}{C_q} \frac{2}{\varrho d V^2} d\varphi. \tag{5.6}$$

By use of relations (5.4) to (5.6) three equations are obtained which represent the change in cable length and the changes in the coordinates of the position of the cable ends under the influence of change in force dT_1 and towing angles $d\phi_1$ and $d\phi_0$ as in [10]: /68

$$\frac{l_1}{T_1} dT_1 + \frac{\tau_1'}{\tau_1} d\varphi_1 \left(\frac{T_1\tau_1}{\sin\varphi_1} \frac{2}{\varrho d V^2 C_q} - \tau_1 l_1 \right) - \frac{T_1\tau_0}{\sin\varphi_0} \frac{2}{\varrho d V^2 C_q} \frac{\tau_0'}{\tau_1} e^{\eta_1-\eta_0} d\varphi_0 = l_1 \lambda dT_1, \tag{5.7}$$

$$\frac{x_1}{T_1} dT_1 + \frac{\tau_1'}{\tau_1} d\varphi_1 \left(\frac{T_1\tau_1 \cos\varphi_1}{\sin\varphi_1} \frac{2}{\varrho d V^2 C_q} - \tau_1 x_1 \right) + \tag{5.8}$$

$$- \frac{T_1\tau_0 \cos\varphi_0}{\sin\varphi_0} \frac{2}{\varrho d V^2 C_q} \frac{\tau_0'}{\tau_1} e^{\eta_1-\eta_0} d\varphi_0 = l_1 \cos\varphi_1 \lambda dT_1 + dx_1, \tag{5.9}$$

after converting equations (5.7) to (5.9) and eliminating $\tau'_0 d\phi_0$ and $\tau'_1 d\phi_1$ we obtain the relations

$$\frac{z_1}{T_1} dT_1 + \frac{\tau_1'}{\tau_1} d\varphi_1 \left(T_1\tau_1 \frac{2}{\varrho d V^2 C_q} - \tau_1 z_1 \right) - T_1\tau_0 \frac{2}{\varrho d V^2 C_q} \frac{\tau_0'}{\tau_1} e^{\eta_1-\eta} d\varphi_0 = l_1 \sin\varphi_1 \lambda dT_1 + dz_1. \tag{5.10}$$

$$dT_1 = \frac{1}{\delta(1 - q l_1 \lambda \sin\varphi_1)} \{ dx_1 [T_1(\sin\varphi_1 - \sin\varphi_0) - q \sin\varphi_1 (z_1 - l_1 \sin\varphi_0)] + \tag{5.11}$$

$$+ dz_1 [T_1(\cos\varphi_0 - \cos\varphi_1) - q \sin\varphi_1 (l_1 \cos\varphi_0 - x_1)]\},$$

where

$$T_1 d\varphi_1 = \frac{q\cos\varphi_1 + n\sin^2\varphi_1}{\delta(1-ql_1\lambda\sin\varphi_1)}\{dx_1[l_1\sin\varphi_0 - z_1 + \lambda T_1 l_1(\sin\varphi_1 - \sin\varphi_0)] + $$
$$+ dz_1[x_1 - l_1\cos\varphi_0 + \lambda T_1 l_1(\cos\varphi_0 - \cos\varphi_1)]\}, \qquad (5.12)$$

Substituting relations (5.10) and (5.11) in equations (5.3) and arranging relative to dx_1 and dz_1, we obtain the cable derivatives in the form:

$$\begin{aligned} X^l_{x_1} &= A_1 X'_{x_1} - A_2 X''_{x_1}, & X^l_{z_1} &= A_1 X'_{z_1} - A_2 X''_{z_1}, \\ Z^l_{x_1} &= A_1 Z'_{x_1} + A_2 Z''_{x_1}, & Z^l_{z_1} &= A_1 Z'_{z_1} + A_2 Z''_{z_1}, \end{aligned} \qquad (5.13)$$

where

$$A_1 = \frac{1}{1-\lambda ql_1\sin\varphi_1}, \quad A_2 = \frac{\lambda T_1}{1-\lambda ql_1\sin\varphi_1}. \qquad (5.14)$$

The terms X'_{x_1}, X'_{z_1}, Z'_{x_1}, and Z'_{z_1} are the cable derivatives calculated by S. Neumark [10] for inextensible cables. When the coefficient of inextensibility $\lambda = 0$, expressions (5.13) for cable derivatives coincide with the results of S. Neumark:

$$\begin{aligned} X'_{x_1} &= \frac{1}{\delta}[T_1\cos\varphi_1(\sin\varphi_1 - \sin\varphi_0) + n\sin^3\varphi_1(z_1 - l_1\sin\varphi_0)], \\ X'_{z_1} &= \frac{1}{\delta}[T_1\cos\varphi_1(\cos\varphi_0 - \cos\varphi_1) + n\sin^3\varphi_1(l_1\cos\varphi_0 - x_1)], \\ Z'_{x_1} &= \frac{1}{\delta}[T_1\sin\varphi_1(\sin\varphi_1 - \sin\varphi_0) - (q + n\sin^2\varphi_1\cos\varphi_1)(z_1 - l_1\sin\varphi_0)], \\ Z'_{z_1} &= \frac{1}{\delta}[T_1\sin\varphi_1(\cos\varphi_0 - \cos\varphi_1) - (q + n\sin^2\varphi_1\cos\varphi_1)(l_1\cos\varphi_0 - x_1)]. \end{aligned} \qquad (5.15)$$

The other terms of the cable derivatives are of the form: /69

$$\begin{aligned} X''_{x_1} &= \frac{l_1}{\delta}\sin\varphi_1(\sin\varphi_1 - \sin\varphi_0)(q\cos\varphi_1 + n\sin^2\varphi_1), \\ X''_{z_1} &= \frac{l_1}{\delta}\sin\varphi_1(\cos\varphi_0 - \cos\varphi_1)(q\cos\varphi_1 + n\sin^2\varphi_1), \\ Z''_{x_1} &= \frac{l_1}{\delta}\cos\varphi_1(\sin\varphi_1 - \sin\varphi_0)(q\cos\varphi_1 + n\sin^2\varphi_1), \\ Z''_{z_1} &= \frac{l_1}{\delta}\cos\varphi_1(\cos\varphi_0 - \cos\varphi_1)(q\cos\varphi_1 + n\sin^2\varphi_1). \end{aligned} \qquad (5.16)$$

It is relatively simple to calculate the cable derivatives by using formulas (5.12)-(5.16). They depend on ϕ_1, ϕ_0, T_1, l_1, x_1, z_1, λ, q, and n, which quantities we calculate on the basis of section 4 of the present paper.

6. Longitudinal Equilibrium of a Glider in Towed Flight

Equilibrium of the forces and moments acting on a glider must occur in steady horizontal towed flight (Figure 6).

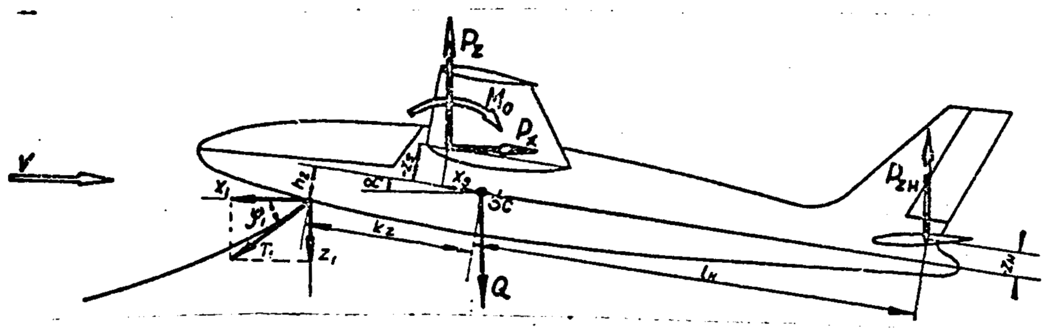

Figure 6. Distribution of Forces and Moments Acting on a Glider in Towed Flight and Geometric Relations

Three equilibrium equations are obtained:

$$P_z+P_{zH}-Q-T_1\sin\varphi_1=0, \tag{6.1}$$

$$T_1\cos\varphi_1-P_x=0, \tag{6.2}$$

$$M_0+P_z(x_s\cos\alpha-z_s\sin\alpha)+P_x(x_s\sin\alpha+z_s\cos\alpha)+P_{zH}(l_H\cos\alpha+z_H\sin\alpha)+ \\ +T_1\cos\varphi_1(h_z\cos\alpha-k_z\sin\alpha)-T_1\sin\varphi_1(h_z\sin\alpha+k_z\cos\alpha)=0. \tag{6.3}$$

After conversion and division by the sides, from equations (6.1) and (6.2) we obtain

$$\mathrm{tg}\varphi_1=\frac{P_z-Q}{P_x}+\frac{P_{zH}}{P_x}. \tag{6.4}$$

The dependence on the aerodynamic lift of the control surfaces P_zH is substituted in equation (6.4) [4]:

$$P_{zH} = \frac{1}{2}\varrho S_H V_H^2 \frac{\partial C_{zH}}{\partial \alpha_H}\left(\alpha - \varepsilon + \alpha_{zH} + \frac{\partial C_{zH}}{\partial \beta_H}\frac{\partial \alpha_H}{\partial C_{zH}}\beta_H\right) \tag{6.5}$$

and the dependence on the aerodynamic lift of the glider P_z and the total resistance P_x are represented by means of the coefficients

$$P_z = \frac{1}{2}\varrho S V^2 C_z, \quad P_x = \frac{1}{2}\varrho S V^2 C_x$$

We obtain the limit of the towing angle ϕ_P due to equilibrium of the forces in extreme displacement of the elevator $+\beta_{H\ max}$ (downward) and $-\beta_{H\ min}$ (upward): /70

$$\operatorname{tg}\varphi_P = \frac{1}{C_x}\left\{C_z\left[1 + \frac{S_H}{S}\left(\frac{V_H}{V}\right)^2 \frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right] + \right.$$
$$\left. + \frac{S_H}{S}\left(\frac{V_H}{V}\right)^2 a_1\left(\alpha_{zH} \pm \frac{a_2}{a_1}\beta_{H\substack{max\\min}}\right) - \frac{2Q}{\varrho S V^2}\right\}, \tag{6.6}$$

where

$$a = \frac{dC_z}{d\alpha}, \quad a_1 = \frac{\partial C_{zH}}{\partial \alpha_H}, \quad a_2 = \frac{\partial C_{zH}}{\partial \beta_H}.$$

The force of tension of the cable arising at the glider towing attachment is determined from equation (6.2):

$$T_1 = \frac{P_x}{\cos\varphi_1}. \tag{6.7}$$

Knowing the glider resistance P_x and the towing angle ϕ_1, we calculate force T_1. From equation (6.3) we determine the force P_zH which must arise on the elevator unit in order to insure equilibrium of the moments in steady flight. After substituting force P_zH as thus calculated in equation (6.4) we obtain the formula for the dependence of the towing angle ϕ_1 on the coefficient of aerodynamic lift of the glider C_z:

$$\operatorname{tg}\varphi_1 = \frac{C_{mbH}\, la + \left(C_z - \frac{2Q}{\varrho S V^2}\right)(l_H \cos\alpha + z_H \sin\alpha) + C_x (h_z \cos\alpha - k_z \sin\alpha)}{C_x[(l_H + k_z)\cos\alpha + (z_H + h_z)\sin\alpha]}, \tag{6.8}$$

where C_{mbH} is the coefficient of the pitching moment of the glider without elevator unit [4] and is of the form:

$$C_{mbH} = C_{m0} + C_z \bar{x}_s + (C_x - C_z \alpha)\bar{z}_s$$

Relation (6.8) defines for the given lift coefficient C_z the only possible towing angle ϕ_1 which ensures equilibrium of the glider. These are the values C_z of towing necessary for towed flight of a given glider at a specific speed.

Because of the limited value of aerodynamic lift $P_z H$ on the control surface corresponding to the extreme displacement of the elevator, by using equations (6.3) and (6.5) we obtain the limits of the towing angle ϕ_M due to equilibrium of the moments:

$$\operatorname{tg}\varphi_M = \frac{C_{mbH}\, la + C_x(h_z \cos\alpha - k_z \sin\alpha) - \frac{S_H}{S}\left(\frac{V_H}{V}\right)^2 a_1 (l_H \cos\alpha + z_H \sin\alpha)\left[\alpha\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \alpha_{zH} \pm \frac{a_2}{a_1}\beta_{H\,\min}^{\;\max}\right]}{C_x(k_z \cos\alpha + h_z \sin\alpha)}.$$

In Figure 7 the C_z of towing is plotted against the towing angle ϕ_1 and curves of the limits due to equilibrium of the forces ϕ_P and due to the equilibrium of the moments ϕ_M are entered in the figure. It is to be seen from the drawing that equilibrium of a glider in towed flight is possible only when the curve of the values C_z of towing is within the range of the available values C_z. /71
This condition may be achieved by suitable location of the towing attachment and proper structural and aerodynamic design of the elevator unit.

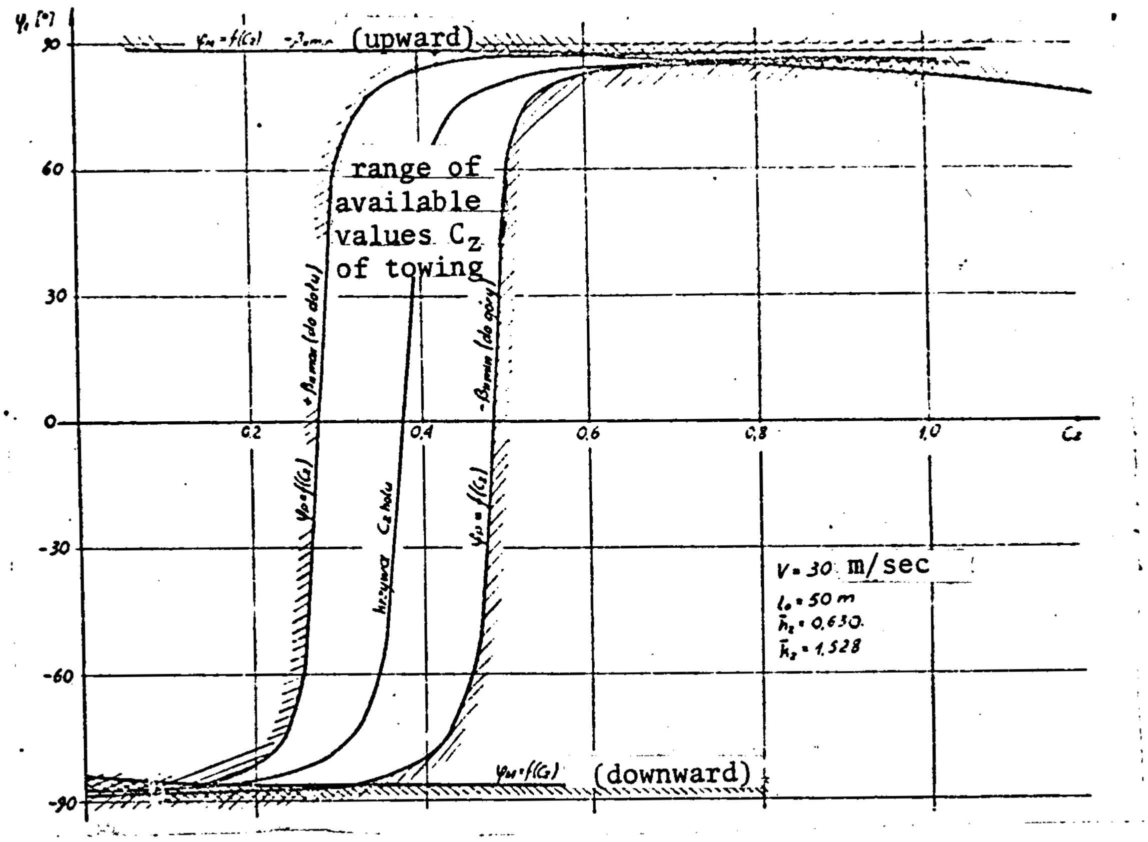

Figure 7. Lift Coefficients Necessary for Towed Flight (values C_z of towing) As a Function of Towing Angle ϕ_1, with Limits Due to Euiilibrium of Forces $\phi_P = f(C_z)$ and Due to Equilibrium of Moments $\phi_M = f(C_z)$ for Extreme Displacements of Elevator

7. Static Longitudinal Stability of a Glider in Towed Flight

The concept of static stability adopted in aviation is discussed at length in papers devoted to the dynamics of aircraft flight [3, 4, 11].

The condition of longitudinal static stability in free flight is

$$\frac{dC_m}{dC_z} < 0 \tag{7.1}$$

for the positive direction of action of the moments adopted in the present paper (the positive moment is the moment lifting the glider and causing increase in the angle of attack).

The static stability has to do only with the occurrence of moments causing return of the glider to a state of equilibrium during the first moment after perturbation of flight [3, 4, 11].

By analogy with free flight it is proposed that the concept of static stability in free flight be introduced, with the definition $C_{mh} = f(C_z)$, i.e., the coefficient of the pitching moment calculated relative to the center of gravity of the glider (Figure 6).

In dimensionless form we represent the pitching moment by means of the coefficient /72

$$C_{mh} = C_{ma} + C_{ml}, \tag{7.2}$$

where

C_{mh} designates the coefficient of pitching moment of a glider in towed flight,

C_{ma} is the coefficient of the pitching moment deriving from the aerodynamic forces acting on the glider,

C_{ml} is the coefficient of the pitching moment deriving from the towing cable.

After differentiation of (7.2) relative to C_z, we obtain

$$\frac{dC_{mh}}{dC_z} = \frac{dC_{ma}}{dC_z} + \frac{dC_{ml}}{dC_z}.$$

In accordance with the definitions adopted in aviation [3, 4, 11] we obtain the static margin in towed flight $\bar{h}_h$ in the form

$$\bar{h}_h = \bar{h}_1 + \bar{h}_l, \tag{7.3}$$

where

$\bar{h}_1 = -\frac{dC_{ma}}{dC_z}$ is the static margin with the control surface held in free flight,

$\bar{h}_l = -\frac{dC_{ml}}{dC_z}$ is the variation in static margin deriving from the towing cable.

The static margin $\bar{h}_1$ in free flight has not been derived in the present paper and has been adopted on the basis of [4] in the final form

$$\bar{h}_1 = \frac{S_H l_H}{S l_a}\left(\frac{V_H}{V}\right)^2 \frac{a_1}{a}\left(1-\frac{d\varepsilon}{d\alpha}\right) - \bar{x}_s - \frac{2C_z}{a}\left(1-\frac{a}{\pi \lambda_e}\right)\bar{z}_s. \tag{7.4}$$

Let us consider the moments of the forces deriving from the towing cable (Figure 6). The components X_1^l and Z_1^l of the force deriving from cable tension and applied to the glider towing attachment vary as a function of the angle of attack α. We determine them by using the cable derivatives derived in section 5 of this paper:

$$\begin{aligned} X_1^l &= T_1\cos\varphi_1 - X_{x_1}^l(h_z\sin\alpha + k_z\cos\alpha - k_z) + X_{z_1}^l(k_z\sin\alpha - h_z\cos\alpha + h_z), \\ Z_1^l &= T_1\sin\varphi_1 - Z_{x_1}^l(h_z\sin\alpha + k_z\cos\alpha - k_z) + Z_{z_1}^l(k_z\sin\alpha - h_z\cos\alpha + h_z). \end{aligned} \tag{7.5}$$

The moment of the forces deriving from the cable is of the form

$$M_l = X_1^l(h_z\cos\alpha - k_z\sin\alpha) - Z_1^l(k_z\cos\alpha + h_z\sin\alpha). \tag{7.6}$$

After substituting relations (7.5) and (6.7) in (7.6) and taking into account the fact that angle of attack α is small

$$\sin\alpha \approx \alpha, \quad \cos\alpha \approx 1 - \frac{\alpha^2}{2}$$

and dividing the equation for the moments by $1/2\rho SV^2 l_a$, we obtain the coefficient of the moment in dimensionless form:

$$\begin{aligned} C_{ml} = &-(\bar{x}_{x_1}\bar{h}_z - \bar{x}_{z_1}\bar{k}_z)(\alpha\bar{h}_z - \alpha^2\bar{k}_z)l_a + (\bar{z}_{x_1}\bar{h}_z - \bar{z}_{z_1}\bar{k}_z)(\bar{k}_z\alpha + \bar{h}_z\alpha^2)l_a + \\ &+ (\bar{x}_{x_1}\bar{k}_z + \bar{x}_{z_1}\bar{h}_z)l_a\bar{h}_z\frac{\alpha^2}{2} - (\bar{z}_{x_1}\bar{k}_z + \bar{z}_{z_1}\bar{h}_z)l_a\bar{k}_z\frac{\alpha^2}{2} + \\ &+ C_x(\bar{h}_z - \bar{k}_z\,\mathrm{tg}\varphi_1) - C_x\alpha(\bar{k}_z + \bar{h}_z\,\mathrm{tg}\varphi_1) + C_x\frac{\alpha^2}{2}(\bar{k}_z\,\mathrm{tg}\varphi_1 - \bar{h}_z), \end{aligned} \tag{7.7}$$

where /73

$$\bar{x}_{x_1} = \frac{X^l_{x_1}}{\frac{1}{2}\varrho S V^2}, \quad \bar{k}_z = \frac{k_z}{l_a}, \quad \bar{h}_z = \frac{h_z}{l_a}$$

$\bar{x}_{z_1}$, $\bar{z}_{x_1}$, and $\bar{z}_{z_1}$ are obtained analogously to $\bar{x}_{x_1}$.

The resistance coefficient C_x and angle of attack α are represented as functions of the lift coefficient C_z [4], and then relation (7.7) is differentiated relative to C_z.

We obtain the variation in the static margin $\bar{h}_l$ deriving from towing:

$$\bar{h}_l = \Delta\bar{h}(l) + \Delta\bar{h}(\bar{k}_z) + \Delta\bar{h}(\bar{h}_z), \tag{7.8}$$

where

$$\Delta\bar{h}(l) = \frac{l_a}{a}\Big\{\bar{h}_z^2\Big[\bar{x}_{x_1} - \frac{C_z}{a}(\bar{x}_{z_1} + 2\bar{z}_{x_1})\Big] + \bar{k}_z^2\Big[\bar{z}_{z_1} + \frac{C_z}{a}(\bar{z}_{x_1} + 2\bar{x}_{z_1})\Big] - \bar{h}_z\bar{k}_z\Big[\bar{x}_{z_1} + \bar{z}_{x_1} + \frac{3C_z}{a}\bar{x}_{x_1} - \bar{z}_{z_1})\Big]\Big\}, \tag{7.9}$$

$$\Delta\bar{h}(\bar{k}_z) = \bar{k}_z\left[\frac{C'_{x_0}}{a} + \frac{3C_z^2}{\pi a \Lambda_e} + \left(\frac{2C_z}{\pi\Lambda_e} - \frac{C_z C_{x_0}}{a^2} - \frac{2C_z^3}{\pi a^2 \Lambda_e}\right)\operatorname{tg}\varphi_1\right], \tag{7.10}$$

$$\Delta\bar{h}(\bar{h}_z) = -\bar{h}_z\left[\frac{2C_z}{\pi\Lambda_e} - \frac{C_z C_{x_0}}{a^2} - \frac{2C_z^3}{\pi a^2 \Lambda_e} - \left(\frac{C_{x_0}}{a} + \frac{3C_z^2}{\pi a \Lambda_e}\right)\operatorname{tg}\varphi_1\right]; \tag{7.11}$$

$\Delta\bar{h}(l)$ indicates the variation in the static stability deriving from the towing cable. It depends on the properties of the cable (unit weight, diameter, extensibility, length), the configuration of the cable, and the position of the towing attachment. This quantity causes increase or decrease in the static margin, depending on the configuration;

$\Delta\bar{h}(\bar{k}_z)$ indicates the variation in the static margin deriving from displacement of the towing attachment horizontally relative to the center of gravity of the glider; it depends on the aerodynamic data of the glider. Displacement of the attachment forward relative to the center of gravity of the glider causes this value to be always positive and produces increase in the static margin;

$\Delta\bar{h}(\bar{h}_z)$ indicates the variation in the static margin caused by displacement of the towing attachment vertically relative to the center of gravity of the glider. Displacement of the attachment below the center of gravity chiefly causes decrease in the static margin.

The static margin in towed flight $\bar{h}_h$ is defined by (7.3) after preceding calculation of $\bar{h}_1$ from (7.4) and $\bar{h}_l$ from (7.8).

As we see, the variations in the static margin caused by towing depend on many factors and may therefore cause increase or decrease in the static stability in relation to the static margin $\bar{h}_1$, corresponding to free flight with the control surface held.

The influence of the individual factors on the longitudinal static margin will be discussed in detail in section 9 of the present paper.

The static margin affects the dynamic stability of a glider. It appears in differential equations of motion as coefficient w of the aerodynamic derivative m_w.

8. Simplified Analysis of Longitudinal Dynamic Stability of a Glider in Towed Flight

The equations of motion of a glider in towed flight have been derived by considering the small perturbations of steady horizontal straight flight; this has permitted linearization of the equations. The linearization has made it possible to obtain solutions in a simple form for more convenient stability analysis.

The small perturbations are designated as follows (Figure 8): /74

x'_1, z'_1 variation in position of center of gravity of glider relative to system x_1,z_1 connected to aircraft,

θ variation in angle of inclination (rotation relative to center of gravity),

u, w components of variations in velocity related to glider in the directions of the axes x and z (related to glider),

q variation in angular velocity of inclination,

u_1, w_1 components of variations in glider speed in directions of axes x_1 and z_1 (related to aircraft).

The equations of motion of a glider in free flight relative to the axes relating to the glider (x, z) are given in [3, 4, 11]. After we introduce into them the forces deriving from the towing cable we obtain

$$
\begin{aligned}
m\dot{u} &= -mg\vartheta\cos\Theta_1 + X_u u + X_w w + X_q q + X^l_{x_1} x'_1 + X^l_{z_1} z'_1 + X^l_{\vartheta}\vartheta, \\
m(\dot{w} - U_1 q) &= mg\vartheta\sin\Theta_1 + Z_u u + Z_w w + Z_q q + Z^l_{x_1} x'_1 + Z^l_{z_1} z'_1 + Z^l_{\vartheta}\vartheta, \\
I_y \dot{q} &= M_u u + M_w w + M_{\dot{w}}\dot{w} + M_q q + M^l_{x_1} x'_1 + M^l_{z_1} z'_1 + M^l_{\vartheta}\vartheta, \\
q &= \dot{\vartheta}.
\end{aligned}
\tag{8.1}
$$

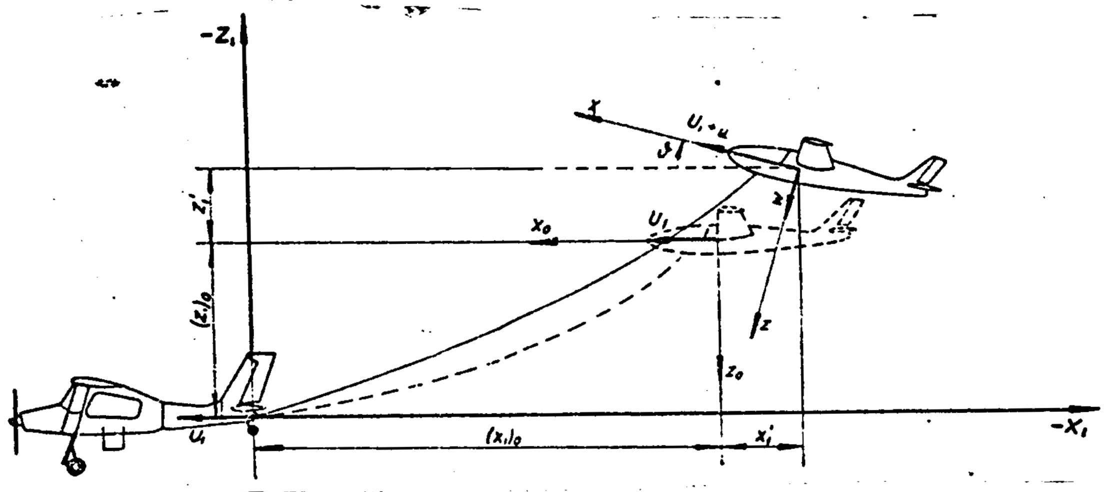

Figure 8. Variation in Glider Position Caused by Perturbation Relative to Straight Horizontal Steady Flight; Interrelations in Coordinate Systems Adopted

It is assumed that the towing unit (aircraft plus glider) were in horizontal steady straight flight, $\Theta_1 = 0$. Taking this assumption into account and using the relations derived in [3], we obtain

$$\dot{x}' = u = u_1, \tag{8.2}$$

$$\dot{z}'_1 = w - U_1\vartheta = w_1. \tag{8.3}$$

converting relation (8.3) we obtain

$$w = \dot{z}_1' + U_1 \vartheta = w_1 + U_1 \vartheta. \tag{8.4}$$

Substituting relations (8.2) and (8.4) in equation (8.1), we obtain a system of ordinary differential equations with constant coefficients:

$$\begin{aligned}
m\dot{u}_1 &= X_u u_1 + X_w (w_1 + U_1 \vartheta) + X_q q + X^l_{x_1} x_1' + X^l_{z_1} z_1' + X^l_\vartheta \vartheta - \mathrm{mg}\,\vartheta, \\
m\dot{w}_1 &= Z_u u_1 + Z_w (w_1 + U_1 \vartheta) + Z_q q + Z^l_{x_1} x_1' + Z^l_{z_1} z_1' + Z^l_\vartheta \vartheta, \\
I_y \dot{q} &= M_u u_1 + M_w (w_1 + U_1 \vartheta) + M_{\dot{w}} (\dot{w}_1 + U_1 q) + M_q q + M^l_{x_1} x_1' + M^l_{z_1} z_1' + M^l_\vartheta \vartheta, \\
q &= \dot{\vartheta}.
\end{aligned}$$

The coefficients of the forces and moments in the case of variations in velocity are termed aerodynamic derivatives [3, 4, 11], and we designate them, for example: /75

$$X_u = \frac{\partial X}{\partial u}, \quad Z_q = \frac{\partial Z}{\partial q}, \quad M_{\dot{w}} = \frac{\partial M}{\partial \dot{w}},$$

and in the case of displacements, are termed cable derivatives (section 5) and [10], e.g.:

$$X^l_{x_1} = \frac{\partial X^l_1}{\partial x_1}, \quad Z^l_{x_1} = \frac{\partial Z^l_1}{\partial x_1}, \quad M^l_\vartheta = \frac{\partial M^l}{\partial \vartheta}.$$

The aerodynamic derivatives appearing in system of equations (8.5) are not derived in the present paper. They are widely discussed and derived in many papers dealing with the dynamics of aircraft flight.

By use of [3] the aerodynamic derivatives in dimensionless form employed in the range of small velocities (the compressibility of air not being taken into account) are cited in final form.

The derivatives of the aerodynamic forces and moments as a function of variation in velocity are:

$$x_u = -\dot{C}_x, \quad x_w = \frac{1}{2}\left(C_z - \frac{\partial C_x}{\partial \alpha}\right),$$
$$z_u = -C_z, \quad z_w = -\frac{1}{2}\left(C_x + \frac{dC_z}{d\alpha}\right), \tag{8.6}$$
$$m_u = C_m^{*} \frac{l_a}{l_H}.$$

The derivatives of the aerodynamic forces as a function of the angle of inclination are:

$$x_\vartheta = \frac{mg\cos\theta_1}{\varrho SV^2} = \frac{mg}{\varrho SV^2}. \tag{8.7}$$

The aerodynamic derivatives as a function of the angular velocity of pitching for simple tapered wings are:

$$x_q = 0{,}6\frac{S_H}{S}\left(C_{zH} - \frac{\partial C_{xH}}{\partial \alpha_H}\right) + \frac{1}{2Sl_H}\int\limits_{-\frac{b}{2}}^{\frac{b}{2}}\left(C_z - \frac{\alpha C_x}{\partial \alpha}\right) l_x dy,$$
$$z_q = -0{,}6\frac{S_H}{S}\left(C_{xH} + \frac{\partial C_{zH}}{\partial \alpha_H}\right) - \frac{1}{2Sl_H}\int\limits_{-\frac{b}{2}}^{\frac{b}{2}}\frac{dC_z}{d\alpha}\left(\frac{3}{4} - \frac{l_k}{l}\right) l^2 dy, \tag{8.8}$$
$$m_q = -0{,}6\frac{S_H}{S}\left(C_{xH} + \frac{\partial C_{zH}}{\partial \alpha_H}\right) - \frac{1}{Sl_H^2}\int\limits_{-\frac{b}{2}}^{\frac{b}{2}}\left[\frac{1}{8}\frac{dC_z}{d\alpha}\left(1 - 2\frac{l_k}{l}\right)^2 + \frac{1}{32}\left(2\pi - \frac{dC_z}{d\alpha}\right)\right] l^2 dy.$$

The aerodynamic derivative of the pitching moment relative to the rate of change in the angle of attack is:

$$m_{\dot{w}} = -\frac{1}{2}\frac{S_H}{S}\frac{\partial C_{zH}}{\partial \alpha_H}\frac{d\varepsilon}{d\alpha}. \tag{8,9}$$

The derivative of the pitching moment relative to the vertical velocity depends both on the aerodynamic properties of the glider and on the configuration and characteristics of the towing cable: /76

$$m_w^h = \frac{1}{2}\frac{l_a}{l_H}\frac{dC_m}{d\alpha}.$$

using (7.2) and (7.3) we obtain

$$\frac{dC_m}{d\alpha} = \frac{dC_z}{d\alpha}\left(\frac{dC_{ma}}{dC_z} + \frac{dC_{ml}}{dC_z}\right) = -\frac{dC_z}{d\alpha}(\bar{h}_1 + \bar{h}_l).$$

Hence

$$m_w^h = m_w + m_w^l,$$

where

$$m_w = -\frac{1}{2}\frac{l_a}{l_H}\frac{dC_z}{d\alpha}\bar{h}_1, \quad m_w^l = -\frac{1}{2}\frac{l_a}{l_H}\frac{dC_z}{d\alpha}\bar{h}_l. \qquad (8.10)$$

the linear derivatives of the forces relative to displacements $X^l_{x_1}$, $X^l_{z_1}$, $Z^l_{x_1}$, and $Z^l_{z_1}$ of the end of the cable have been derived in section 5 of the present paper.

The linear derivatives of the pitching moment relative to the displacements and the angle of inclination and the linear derivatives of the forces relative to the angle of inclination are derived below.

The variations in the forces and moments deriving from the cable as a function of the displacement and variation in the angle of inclination are of the form

$$\begin{aligned} dX_1^l &= X^l_{x_1}dx_1 + X^l_{z_1}dz_1 + X^l_\vartheta d\vartheta, \\ dZ_1^l &= Z^l_{x_1}dx_1 + Z^l_{z_1}dz_1 + Z^l_\vartheta d\vartheta, \\ dM_1^l &= M^l_{x_1}dx_1 + M^l_{z_1}dz_1 + M^l_\vartheta d\vartheta. \end{aligned}$$

allowing for the variation in the angle of inclination $d\vartheta$, we obtain (Figure 6)

$$
\begin{aligned}
dX_1^l &= X_{x_1}^l dx_1 + X_{z_1}^l dz_1 (X_{x_1}^l h_z - X_{z_1}^l k_z + T_1 \sin\varphi_1) d\vartheta, \\
dZ_1^l &= Z_{x_1}^l dx_1 + Z_{z_1}^l dz_1 + (Z_{x_1}^l h_z - Z_{z_1}^l k_z - T_1 \cos\varphi_1) d\vartheta, \\
dM_1^l &= dX_1^l h_z - dZ_1^l k_z = (X_{x_1}^l h_z - Z_{x_1}^l k_z) dx_1 + (X_{x_1}^l h_z - Z_{z_1}^l k_z) dz_1 + \\
&\quad + [X_{x_1}^l h_z^2 - (X_{z_1}^l + Z_{x_1}^l) h_z k_z + Z_{z_1}^l k_z^2 + T_1 (h_z \sin\varphi_1 + k_z \cos\varphi_1)] d\vartheta.
\end{aligned}
$$

the cable derivatives in the above equations are the coefficients of dx_1, dz_1, and $d\vartheta$.

The cable derivatives are given below in dimensionless form:

$$
\begin{aligned}
&x_{x_1} = \frac{X_{x_1}^l l_H}{\varrho S V^2}, \quad x_{z_1} = \frac{X_{z_1}^l l_H}{\varrho S V^2}, \quad x_{\vartheta l} = \frac{1}{\varrho S V^2} (X_{x_1}^l h_z - X_{z_1}^l k_z + T_1 \sin\varphi_1), \\
&z_{x_1} = \frac{Z_{x_1}^l l_H}{\varrho S V^2}, \quad z_{z_1} = \frac{Z_{z_1}^l l_H}{\varrho S V^2}, \quad z_\vartheta = \frac{1}{\varrho S V^2} (Z_{x_1}^l h_z - Z_{z_1}^l k_z - T_1 \cos\varphi_1), \\
&m_{x_1} = \frac{1}{\varrho S V^2} (X_{x_1}^l h_z - Z_{x_1}^l k_z), \\
&m_{z_1} = \frac{1}{\varrho S V^2} (X_{z_1}^l h_z - Z_{x_1}^l k_z), \\
&m_\vartheta = \frac{1}{\varrho S V^2 l_H} [X_{x_1}^l h_z^2 - (X_{z_1}^l + Z_{x_1}^l) h_z k_z + Z_{z_1}^l k_z^2 + T_1 (h_z \sin\varphi_1 + k_z \cos\varphi_1)].
\end{aligned}
\tag{8.11}
$$

All aerodynamic and linear derivatives being known, we proceed to solve system of equations (8.5). System of equations (8.5) is converted to dimensionless form by dividing the equations of forces by $\rho V^2 S$, and the equation for the moments by $\rho V^2 S l_H$, and by introducing the following expressions in accordance with the terms adopted in aviation [3, 12, 13]: /77

$\hat{t} = \frac{m}{\varrho V S}$ aerodynamic time,

$\frac{H_{l\varrho}}{m} = \mu$ relative density of aircraft,

$\bar{t} = \frac{t}{\hat{t}}$ dimensionless time,

$j_y = \frac{I_y}{m l^2_H}$ dimensionless moment of inertia,

$\bar{u}_1 = \frac{u_1}{V}$; $\bar{w}_1 = \frac{w_1}{V}$ dimensionless linear velocities,

$\bar{q} = q\hat{t}$ dimensionless angular velocity of pitching,

$$\bar{m}_u = -\frac{\mu_1 m_u}{j_y}, \quad \bar{m}_w^h = -\frac{\mu_1 m_w^h}{j_y}, \quad \bar{m}_q = -\frac{m_q}{j_y},$$

$$\bar{m}_{\dot{w}} = -\frac{\mu_1 m_{\dot{w}}}{j_y}, \quad \bar{m}_{x_1} = -\frac{\mu_1 m_{x_1}}{j_y}, \quad \bar{m}_{z_1} = -\frac{\mu_1 m_{z_1}}{j_y}, \quad \bar{m}_\vartheta = -\frac{\mu_1 m_\vartheta}{j_y}.$$

It has also been assumed that the axes related to the glider have been selected so that the direction of velocity V before the perturbation of the equilibrium agreed with the direction of the x axis selected, i.e., $U_1 = V$. A system of equations in dimensionless form is obtained:

$$\begin{gathered}
\frac{d\bar{u}_1}{d\bar{t}} - x_u\bar{u}_1 - x_w\bar{w}_1 - \frac{x_q}{\mu_1}\bar{q} - x_{x_1}\bar{x}_1 - x_{z_1}\bar{z}_1 + (x_\vartheta - x_w - x_{\vartheta l})\vartheta = 0, \\
-z_u\bar{u}_1 + \frac{d\bar{w}_1}{d\bar{t}} - z_w\bar{w}_1 - \frac{z_q}{\mu_1}\bar{q} - z_{x_1}\bar{x}_1 - z_{z_1}\bar{z}_1 - (z_w + z_\vartheta)\vartheta = 0, \\
\bar{m}_u\bar{u}_1 + \bar{m}_{\dot{w}}\frac{d\bar{w}_1}{d\bar{t}} + \bar{m}_w^h\bar{w}_1 + \frac{d\bar{q}}{d\bar{t}} + \left(\bar{m}_q + \frac{\bar{m}_{\dot{w}}}{\mu_1}\right)\bar{q} + \bar{m}_{x_1}\bar{x}_1 + \bar{m}_{z_1}\bar{z}_1 + (\bar{m}_w^h + \bar{m}_\vartheta)\vartheta = 0, \\
\bar{q} = \frac{d\vartheta}{d\bar{t}}, \quad \bar{u}_1 = \frac{d\bar{x}_1}{d\bar{t}}, \quad \bar{w}_1 = \frac{d\bar{z}_1}{d\bar{t}}.
\end{gathered} \tag{8.12}$$

The general solution of the system of equations is anticipated in the form

$$\bar{x}_1 = x_0 e^{\bar{\lambda}\bar{t}}, \quad \bar{z}_1 = z_0 e^{\bar{\lambda}\bar{t}}, \quad \vartheta = \vartheta_0 e^{\bar{\lambda}\bar{t}}.$$

After substituting the above relations in system of equations (8.12), dividing by $e^{\overline{\lambda t}}$, and ordering relative to x_0, z_0, and θ_0, we obtain a system of homogeneous equations. The condition of solution of this system is that the determinant of the coefficients of x_0, z_0, and $\theta_0 = 0$. After expanding the determinant and ordering relative to the powers of $\bar{\lambda}$, we obtain the characteristic equation in the form

$$\bar{\lambda}^6 + B_1\bar{\lambda}^5 + (C_1 + C_1^l)\bar{\lambda}^4 + (D_1 + D_1^l)\bar{\lambda}^3 + (E_1 + E_1^l)\bar{\lambda}^2 + F_1^l\bar{\lambda} + G_1^l = 0, \tag{8.13}$$

where coefficients B_1, C_1, D_1, and E_1 are the coefficients of a characteristic quadratic equation in the case of free glider flight [3].

The coefficients of characteristic equation (8.13) could be separated into the portion corresponding to free flight and the portion allowing for the /78 influence of towing.

The coefficients of the characteristic equation for free flight are

$$B_1 = B = -(x_u + z_w) + \overline{m}_q + (1 + z_q)\frac{\overline{m}_{\dot{w}}}{\mu_1},$$

$$C_1 = x_u z_w - x_w z_u - (x_u + z_w)\overline{m}_q + \left(-z_w + \frac{x_u + z_w}{\mu_1} + \frac{x_u z_q - x_q z_u}{\mu_1}\right)\overline{m}_{\dot{w}} + \\ + \left(1 + \frac{z_q}{\mu_1}\right)\overline{m}_w + \frac{x_q}{\mu_1}\overline{m}_u,$$

$$D_1 = (x_u z_w - x_w z_u)\overline{m}_q + \left[\left(\frac{1}{\mu_1} - 1\right)(x_u z_w - x_w z_u) - z_u x_\vartheta\right]\overline{m}_{\dot{w}} - \\ - \left(x_u + \frac{x_u z_q - x_q z_u}{\mu_1}\right)\overline{m}_w + \left(x_w - x_\vartheta + \frac{x_w z_q - x_q z_w}{\mu_1}\right)\overline{m}_u,$$

$$E_1 = (z_w \overline{m}_u - z_u \overline{m}_w)x_\vartheta.$$

The variations in the coefficients of the characteristic equation caused by towing are:

$$C_1^l = \left(1 + \frac{z_q}{\mu_1}\right)\overline{m}_w^l - x_{x_1} - z_{z_1} + \overline{m}_\vartheta + z_\vartheta \overline{m}_{\dot{w}},$$

$$D_1^l = x_u z_{z_1} - x_{z_1} z_u + x_{x_1} z_w - x_w z_{x_1} - (x_{x_1} + z_{z_1})\overline{m}_q - \\ - (x_u + z_w)\overline{m}_\vartheta + \left(x_{\vartheta l} z_u - x_u z_\vartheta - \frac{x_{x_1} + z_{z_1}}{\mu_1} + \frac{x_q z_{x_1} - x_{x_1} z_q}{\mu_1}\right)\overline{m}_{\dot{w}} + \\ + z_\vartheta \overline{m}_w^h + x_{\vartheta l}\overline{m}_u + \frac{x_q}{\mu_1}\overline{m}_{x_1} + \frac{z_q}{\mu_1}\overline{m}_{z_1} - \left(x_u + \frac{x_u z_q - x_q z_u}{\mu_1}\right)\overline{m}_w^l,$$

$$E_1^l = x_{x_1} z_{z_1} - x_{z_1} z_{x_1} + [x_{x_1} z_w - x_w z_{z_1} + (x_u - z_u) z_{z_1}]\overline{m}_q + \\ + (x_u z_w - x_w z_u - x_{x_1} - z_{z_1})\overline{m}_\vartheta + \left[\frac{x_u z_{z_1} - x_{z_1} z_u}{\mu_1} + \right. \\ \left. + (x_{x_1} z_w - x_w z_{x_1})\left(\frac{1}{\mu_1} - 1\right) - z_\vartheta x_{x_1} - (x_\vartheta - x_{\vartheta l}) z_{x_1}\right]\overline{m}_{\dot{w}} + \\ + \left(\frac{x_q z_{x_1} - x_{x_1} z_q}{\mu_1} - x_u z_\vartheta + z_u x_{\vartheta l} - x_{x_1} - z_{z_1}\right)\overline{m}_w^h + \left(\frac{x_{z_1} z_q - x_q z_{z_1}}{\mu_1} - x_{\vartheta l} z_u + x_w z_\vartheta\right)\overline{m}_u +$$

$$+\left(\frac{x_w z_q - x_q z_w}{\mu_1} + x_w - x_\vartheta - x_{\vartheta l}\right)\overline{m}_{x_1} + \left(\frac{z_u x_q - x_u z_q}{\mu_1} + z_w + z_\vartheta\right)\overline{m}_{z_1}.$$

$$F_1^l = F = (x_{x_1} z_{z_1} - x_{z_1} z_{x_1})\left(\overline{m}_q + \frac{\overline{m}_{\dot{w}}}{\mu_1}\right) + (x_u z_{z_1} - x_{z_1} z_u + x_{x_1} z_w - x_w z_{x_1})\overline{m}_\vartheta +$$
$$+ [x_u z_{z_1} - x_{z_1} z_u - z_\vartheta x_{x_1} - (x_\vartheta - x_{\vartheta l}) z_{x_1}]\overline{m}_w^h + [z_w x_{z_1} - z_{z_1} x_w + z_\vartheta x_{z_1} - (x_\vartheta - x_{\vartheta l}) z_{z_1}]\overline{m}_u +$$
$$+ \left[\frac{x_{z_1} z_q - x_q z_{z_1}}{\mu_1} - z_\vartheta x_w + (x_\vartheta - x_{\vartheta l}) z_w\right]\overline{m}_{x_1} +$$
$$+ \left[\frac{z_{x_1} x_q - z_q x_{x_1}}{\mu_1} - z_\vartheta x_u - (x_\vartheta - x_{\vartheta l}) z_u + x_w z_u - x_u z_w\right]\overline{m}_{z_1}.$$

$$G_1^l = G = (x_{x_1} z_{z_1} - x_{z_1} z_{x_1})(\overline{m}_\vartheta + \overline{m}_w^h) + [x_{z_1} z_w - x_w z_{z_1} + z_\vartheta x_{z_1} + (x_\vartheta - x_{\vartheta l}) z_{z_1}]\overline{m}_{x_1} +$$
$$+ [z_{x_1} x_w - z_w x_{x_1} - z_\vartheta x_{x_1} - (x_\vartheta - x_{\vartheta l}) z_{x_1}]\overline{m}_{z_1}.$$

The Routh-Hurwitz stability criteria relating to small perturbations of steady movement set the condition that all coefficients of characteristic equation (8.13) be positive:

$$B_1, C, D, E, F, G > 0 \tag{8.14}$$

as well as that the Routh discriminant be greater than zero [1]:

$$R \equiv \Delta_0 \Delta_2 - \Delta_1^2 > 0, \tag{8.15}$$

where

$$\Delta_0 = \begin{vmatrix} B & 1 & 0 \\ D & C & B \\ F & E & D \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} B & 1 & 0 \\ D & C & B \\ 0 & G & F \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} B & 1 & 0 \\ F & E & D \\ 0 & G & F \end{vmatrix}$$

where

$$B = B_1, \quad C = C_1 + C_1^l, \quad D = D_1 + D_1^l, \quad E = E_1 + E_1^l, \quad F = F_1^l, \quad G = G_1^l,$$

To determine whether the glider is stable in towed flight it suffices to check the Routh-Hurwitz criteria of (8.14) and (8.15).

In the case of the solution of equation (8.13) the roots obtained are of the form

$$\bar{\lambda}_n^h = \bar{\xi}_n^h \pm i\bar{\eta}_n^h \tag{8.16}$$

and for a stable glider the damping coefficients are $\overline{\xi_n^h} < 0$, i.e., the motion is damped and the glider is dynamically stable.

The roots of the characteristic equation (eigenvalues) being known, we can calculate the eigen vectors using two equations of system (8.12) in the form

$$\begin{aligned} &(\bar{\lambda}_n^2 - x_u\bar{\lambda}_n - x_{x_1})\bar{x}_1^n - (x_w\bar{\lambda}_n + x_{z_1})\bar{z}_1^n - \left(\frac{x_q}{\mu_1}\bar{\lambda}_n - x_3 + x_w + x_{31}\right)\vartheta^n = 0, \\ &-(z_u\bar{\lambda}_n + z_{x_1})\bar{x}_1^n + (\bar{\lambda}_n^2 - z_w\bar{\lambda}_u - z_{z_1})\bar{z}_1^n - \left(\frac{z_q}{\mu_1}\bar{\lambda}_n + z_w + z_3\right)\vartheta^n = 0, \end{aligned} \tag{8.17}$$

where $\bar{\lambda}_n$ are the corresponding eigen values, and $\bar{x}_1^n$, $\bar{z}_1^n$, and θ^n are the eigen vectors corresponding to them.

We obtain the equations describing the movement of the glider after perturbation in the form

$$\begin{bmatrix} \bar{x}_1 \\ \bar{z}_1 \\ \vartheta \end{bmatrix} = \sum_{n=1}^{6} K_n \begin{bmatrix} \bar{x}_1^n \\ \bar{z}_1^n \\ \vartheta^n \end{bmatrix} e^{\bar{\lambda}_n \bar{t}}. \tag{8.18}$$

The constants K_n are calculated from the initial conditions for $\bar{t} = 0$.

9. Numerical Example and Conclusions

As an example calculations have been made for a prototype high-performance glider. In the calculations the towing parameters have been changed in succession so that it will be possible to find the influence of the individual factors on the stability of the glider in towed flight. At the same time, calculations have been made for stability in free flight and compared with the results of calculations for towed flight. A type C cable, the one the most frequently employed in practice, has been adopted for the calculations. /80

In the diagrams the variations in the coefficients $\bar{\xi}$ of damping have been drawn in continuous lines, and the variation in velocity $\bar{\eta}$ as a function of the towing parameters in broken lines. The thick continuous lines and the broken lines characterize the towed flight of a glider, and the thin lines the free flight of a glider under conditions equivalent to towed flight.

All calculations were performed by a GIER digital computer in accordance with programs in GIER-ALGOL III language. The characteristic sixth-order equation was solved by the Bairstow method [15], this making it possible to find the complex roots.

9.1. *Influence of position of glider relative to towing aircraft on stability.* The glider is towed on a cable of a length of l_0 = 50 m at a constant speed of V = 30 m/sec. The glider may be displaced in the vertical plane z_1 = ±20 m relative to the line of flight of the towing aircraft. The variation in the static margin is shown in Figure 9 calculated from (7.4), (7.8), and (7.3).

After solution of characteristic equation (8.13) six roots were obtained (8.16), which constitute the eigenvalues of the system. There appear two complex conjugate roots $\bar{\lambda}_1^h = \bar{\xi}_1^h \pm i\bar{\eta}_1^h$, $\bar{\lambda}_2^h = \bar{\xi}_2^h \pm i\bar{\eta}_2^h$, corresponding to periodic motion of the glider and two real roots $\bar{\lambda}_3^h = \bar{\xi}_3^h$ i $\bar{\lambda}_4^h = \bar{\xi}_4^h$, corresponding to aperiodic motion. The variation in the eigenvalues is shown in Figure 10 plotted as a function of the position.

By way of example the eigen vectors calculated from equations (8.17) for a selected towing angle of ϕ_1 = 20° are given below; this has permitted proper determination of the natural forms of the system.

$\bar{\lambda}_n^h$	$-2{,}550 \pm i\,2{,}541$	$0{,}168 \pm i\,0{,}216$	$-0{,}256$	$-0{,}017$
x_1^n $\bar{z}_1^n$ ϑ^n	$1 \pm i$ $0{,}046 \pm i\,2{,}650$ $0{,}476 + i\,415$	$1 \pm i$ $2{,}105 \pm i\,0{,}641$ $0{,}385 \pm i\,33{,}200$	1 $-0{,}121$ 0,003	1 22,800 0,411

The eigenvalues $\overline{\lambda}_1^h$ correspond to rapid strongly damped oscillations (high oscillation frequency) about the center of gravity of the glider. The eigenvalues $\overline{\lambda}_3^h$ correspond to aperiodic weakly damped longitudinal displacements of the glider, while eigenvalues $\overline{\lambda}_4^h$ characterize the aperiodic very weakly damped vertical displacements of the glider.

Eigenvalue $\overline{\lambda}_2^h$ characterizes periodic phugoid movements [3] and [4] (of small frequency, weakly damped). It affects all three types of movement. It exerts the strongest effect on oscillations about the center of gravity, and the least effect on horizontal displacements, Calculation of variations in $\overline{x}$, $\overline{z}$, and θ as a function of time (8.18), in relation to the initial conditions indicates that the greatest influence on glider stability in towed flight is exerted by perturbations causing elevation of the glider. The horizontal displacements exert a strong influence on oscillations immediately after the perturbation, while the vertical displacements manifest themselves after a certain period (gradual increase in amplitude) and are weak.

The variation in the position of the glider relative to the line of flight of the towing aircraft has a decisive effect on the phugoid oscillations (eigenvalues $\overline{\lambda}_2^h$) and aperiodic longitudinal movements of the glider ($\overline{\lambda}_3^h$) (Figure 10). In flight of the glider below the line of flight of the towing aircraft, damped phugoid movements ($\overline{\xi}_2^h < 0$) and undamped longitudinal oscillations ($\overline{\xi}_3^h > 0$)occur, /83

Variation in the position of the glider has no effect on rapid oscillations ($\overline{\xi}_1^h$, and $\overline{\eta}_1^h$) are on aperiodic vertical displacements ($\overline{\xi}_4^h$).

Comparing the eigenvalues characterizing the free flight of a glider we see (Figure 10) that towing exerts no effect at all on rapid oscillations ($\overline{\xi}_1^h = \overline{\xi}_1$ and $\overline{\eta}_1^h = \overline{\eta}_1$), while it exerts a strong influence on the damping of phugoid oscillations ($\overline{\xi}_2^h \neq \overline{\xi}_2$) in the case of slight frequency changes ($\overline{\eta}_2^h \approx \overline{\eta}_2$). In towed flight, as compared to free flight, two additional eigenvalues occur, $\overline{\lambda}_3$ and $\overline{\lambda}_4$, which are predominantly real quantities and characterize aperiodic movements.

Elevation of the glider above the towing aircraft causes increase in the static margin, Figure 9, but does not cause increase in the dynamic stability.

9.2. *Influence of towing speed on glider stability.* The glider is towed on a type C cable of a length of l_0 = 50 m at varying speeds.

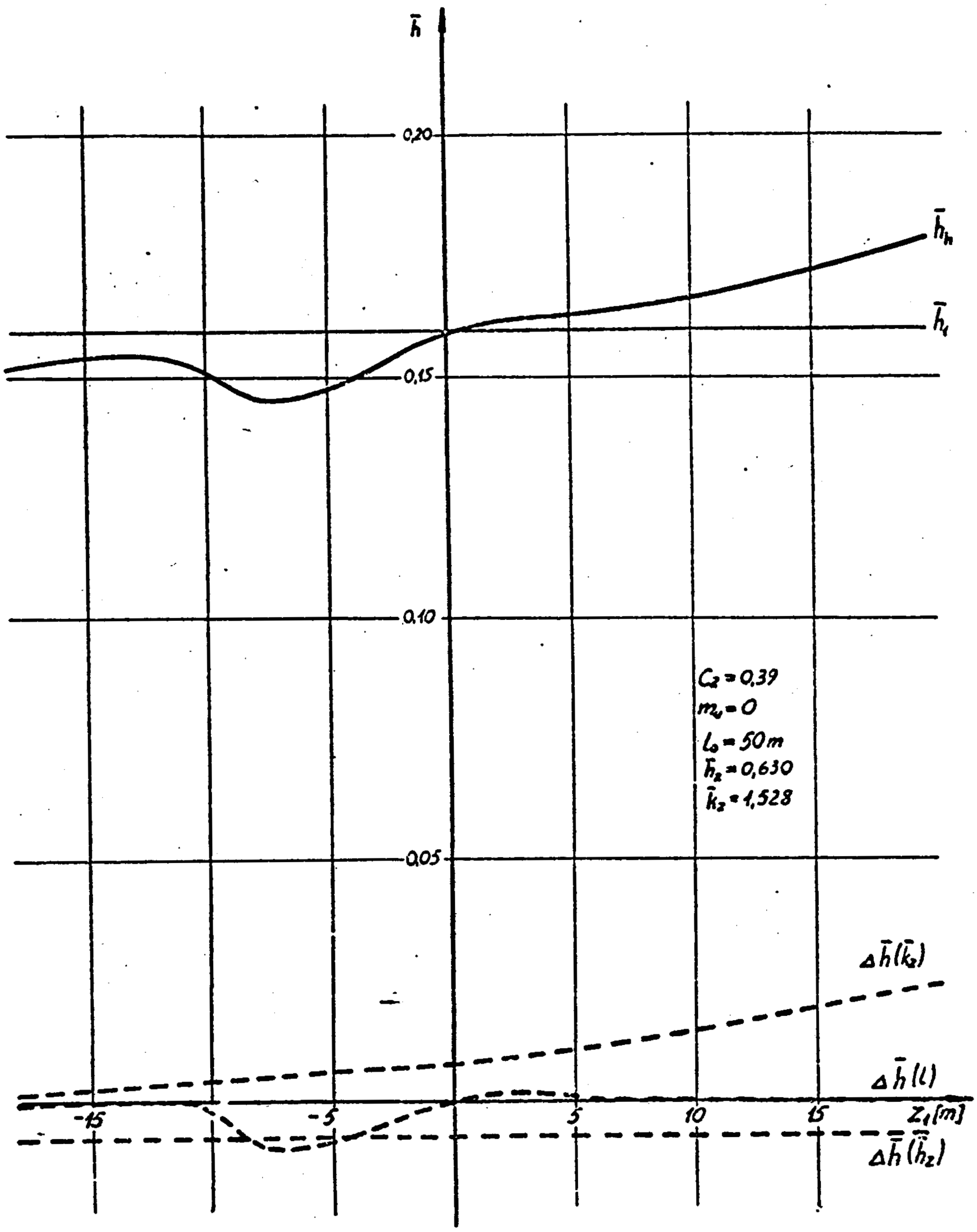

Figure 9. Variation in "Static Margin" of a Glider As a Function of Vertical Position Relative to Towing Aircraft

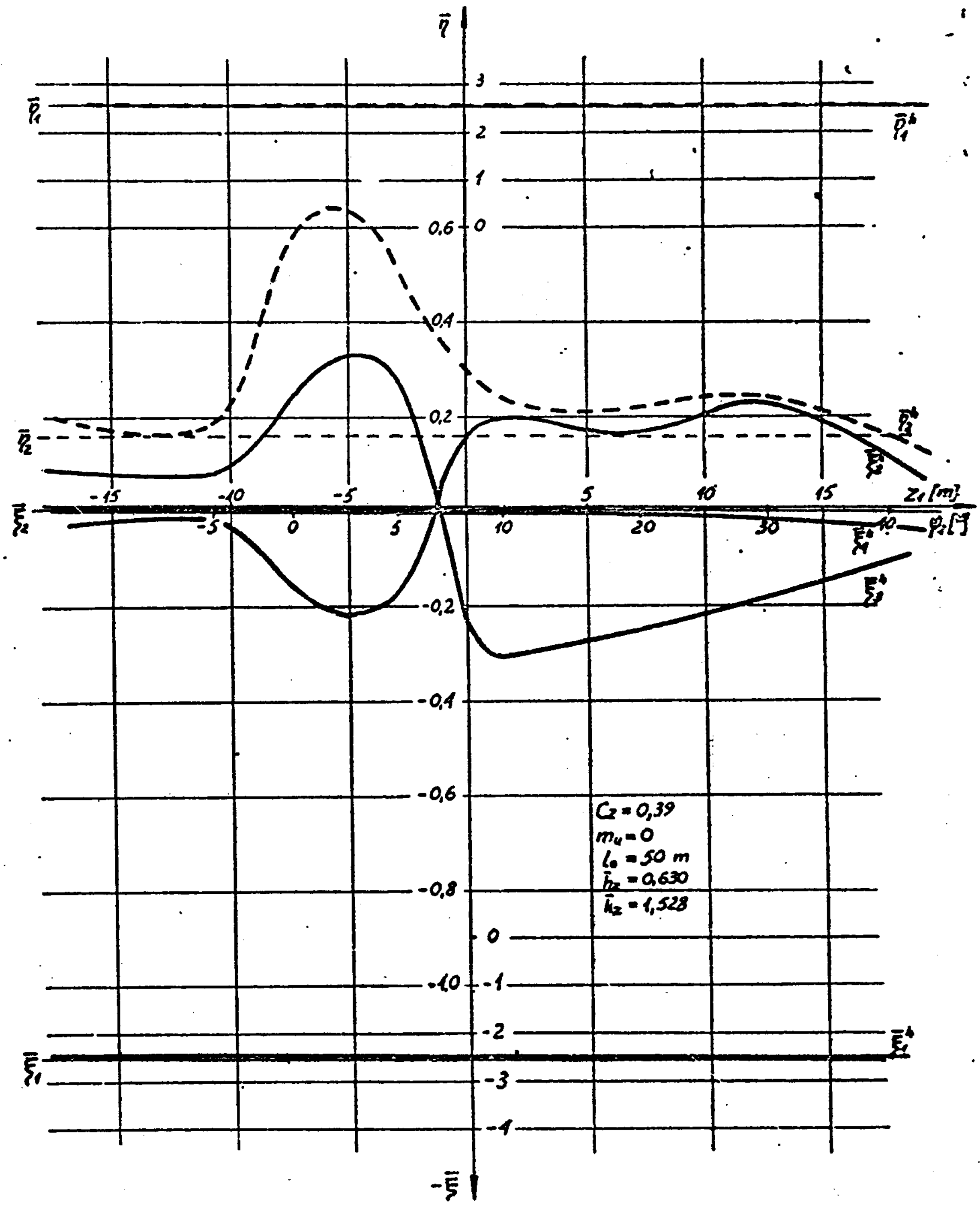

Figure 10. Variation in Coefficients of Damping $\bar{\xi}$ and Frequency $\bar{\eta}$ As a Function of Vertical Position Relative to Towing Aircraft

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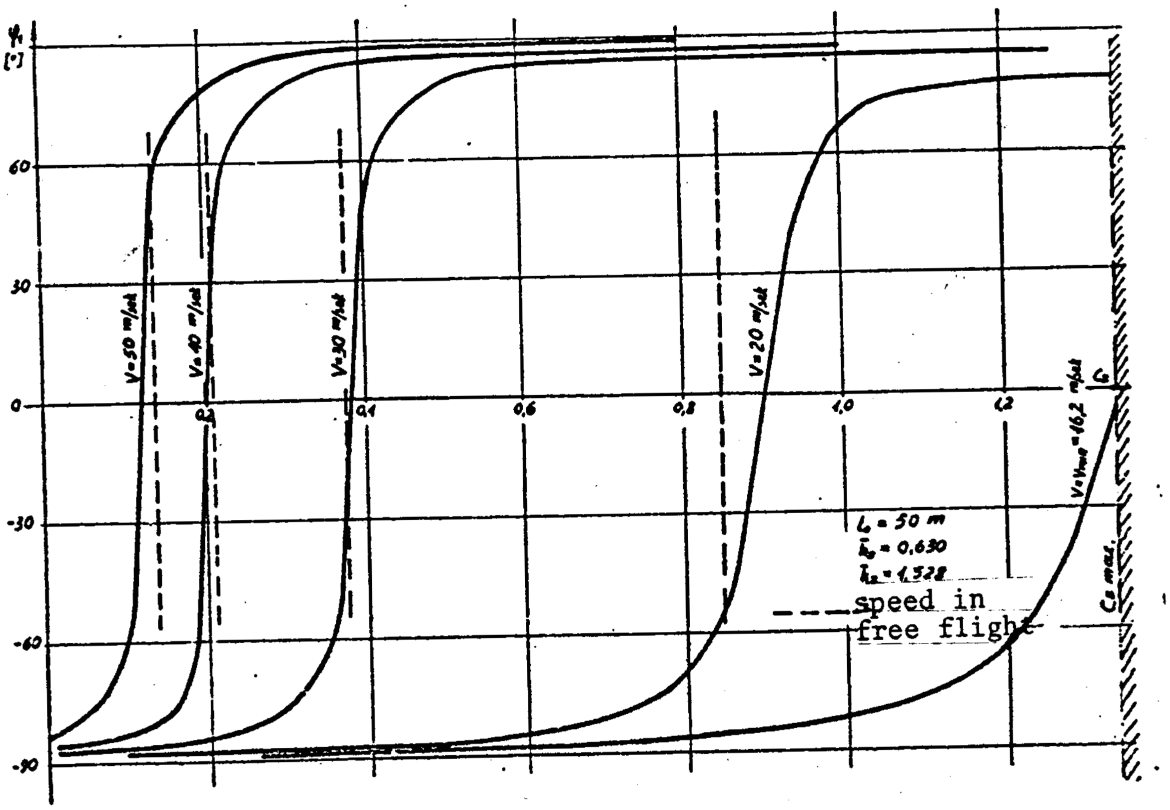

Figure 11. Curves of Glider Equilibrium – Variation in Towing Angle ϕ_1, As a Function of Lift Coefficient C_z for Various Towing Speeds

The variation in the towing angle ϕ_1 as a function of the lift coefficient of the glider C_z has been calculated from (6.8) for several towing speeds. The equilibrium diagrams are given in Figure 11.

We see from Figure 11 that a glider may be towed at a certain speed and in a certain position relative to the towing aircraft (ϕ_1) only at one value of C_z assuring equilibrium. /89

In towed flight the minimum speed V_{min} of a glider corresponding to free flight can be reached only in a position below the line of flight of the towing aircraft. In positions above the line of flight of the towing aircraft *mushing* of the glider ensues at a speed higher than the minimum speed, $V_{minh} > V_{min}$.

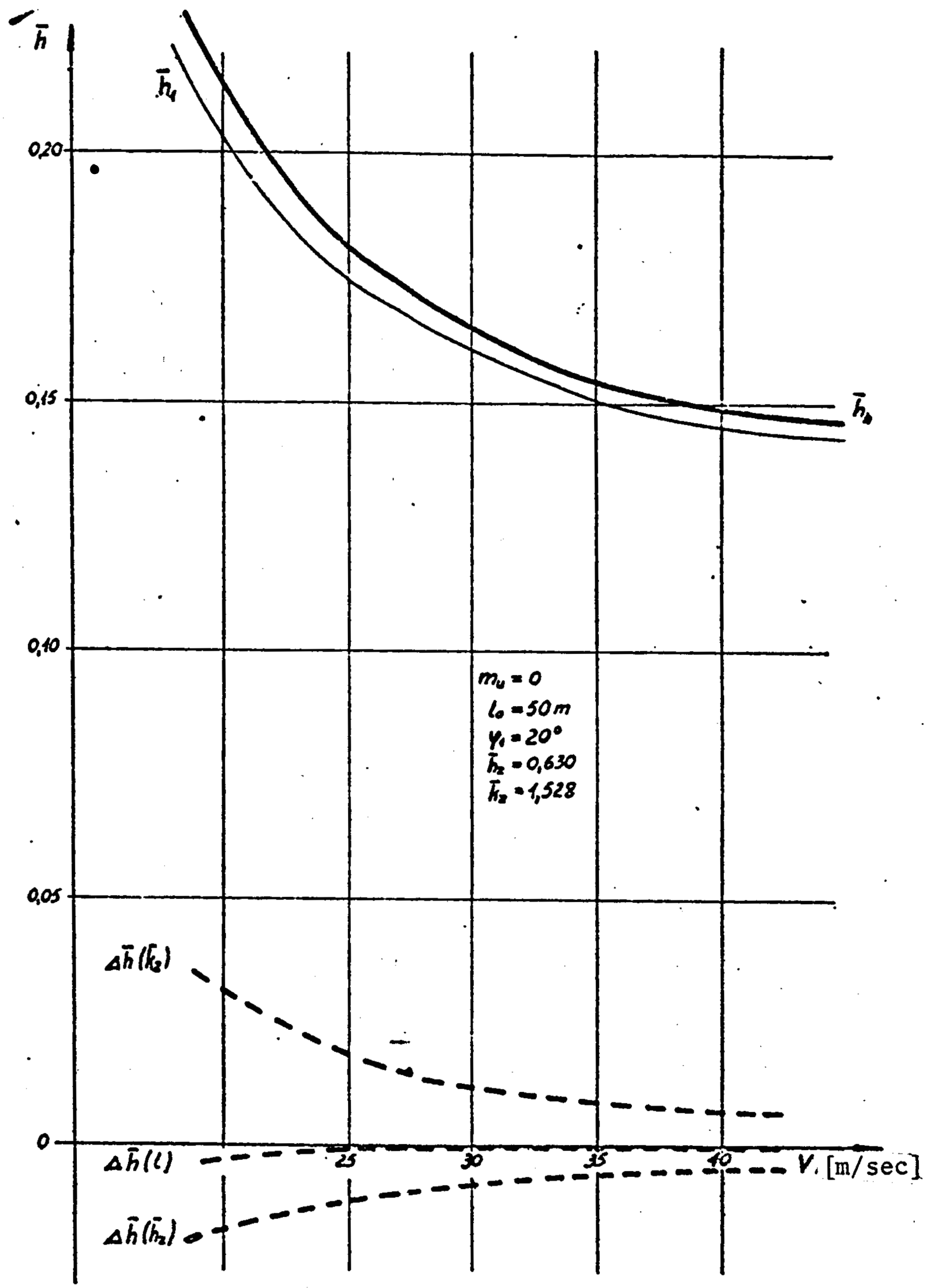

Figure 12. Variation in "Static Margin" of a Glider with Towing Speed for a Towing Angle of $\phi_1 = 20°$

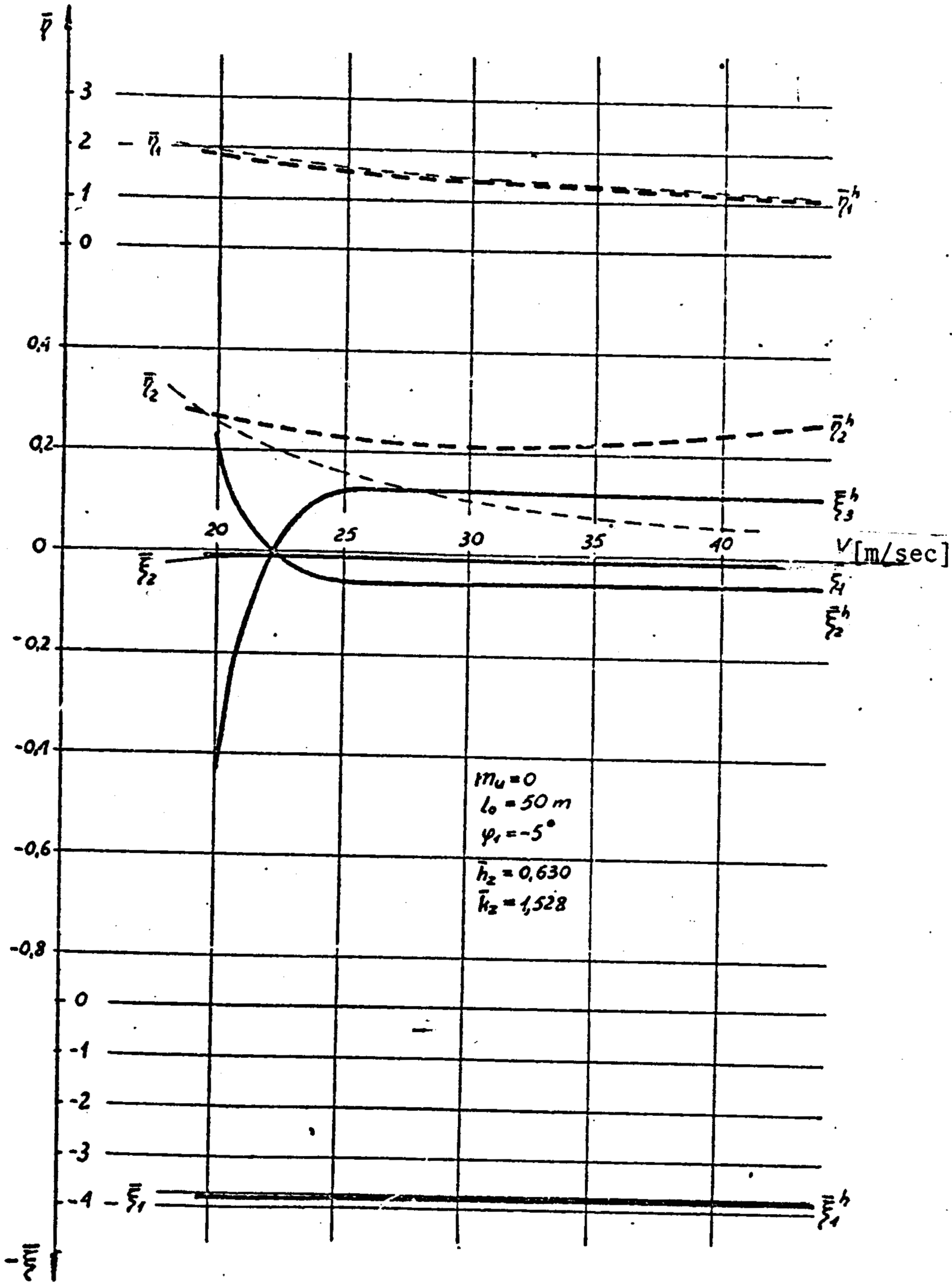

Figure 13. Variation in Damping and Frequency Coefficients of a Glider with Towing Speed for a Towing Angle of $\phi_1 = 20°$

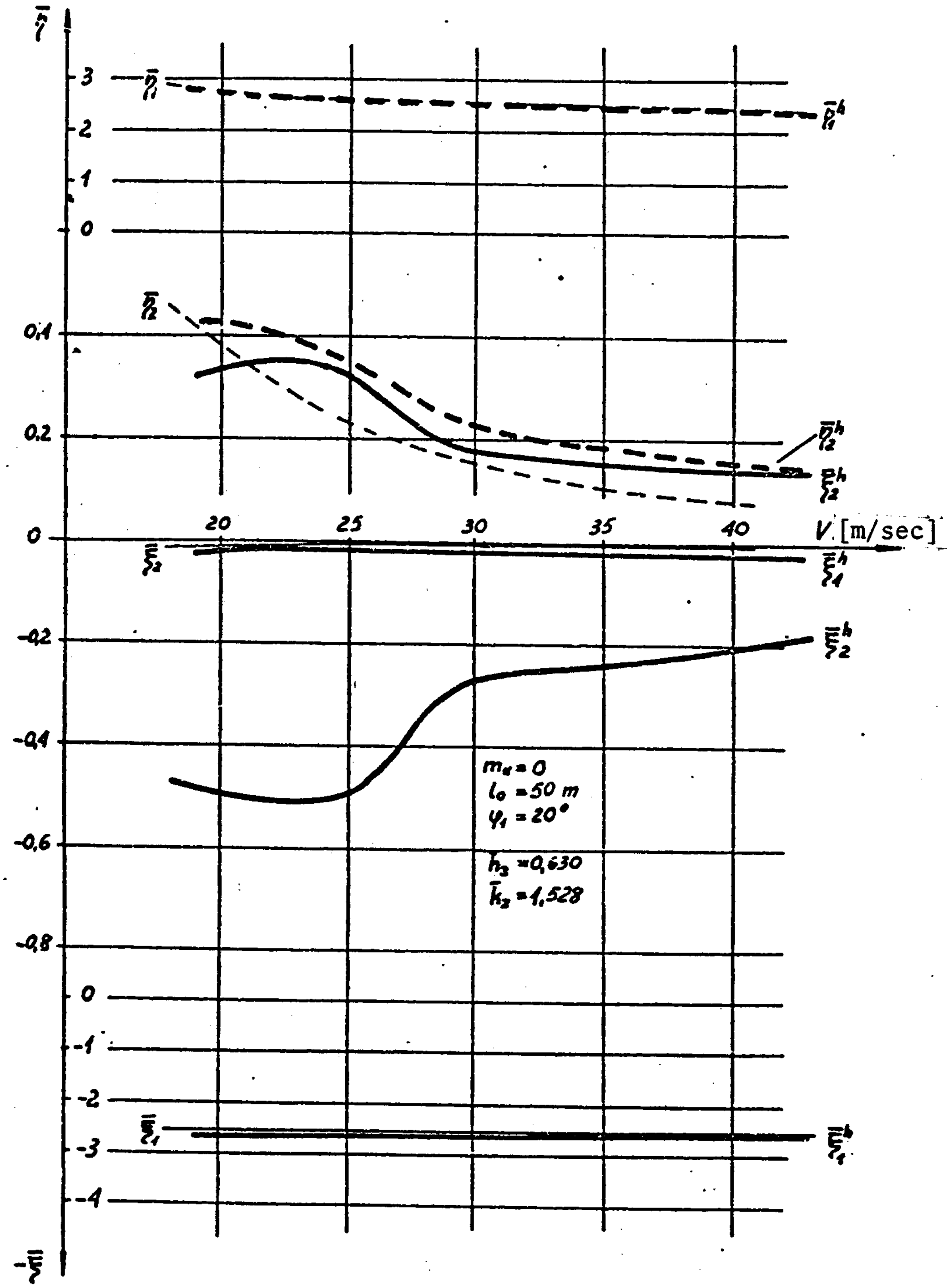

Figure 14. Variation in Damping and Frequency Coefficients of a Glider with Towing Speed for a Towing Angle of ϕ_1 = -5°

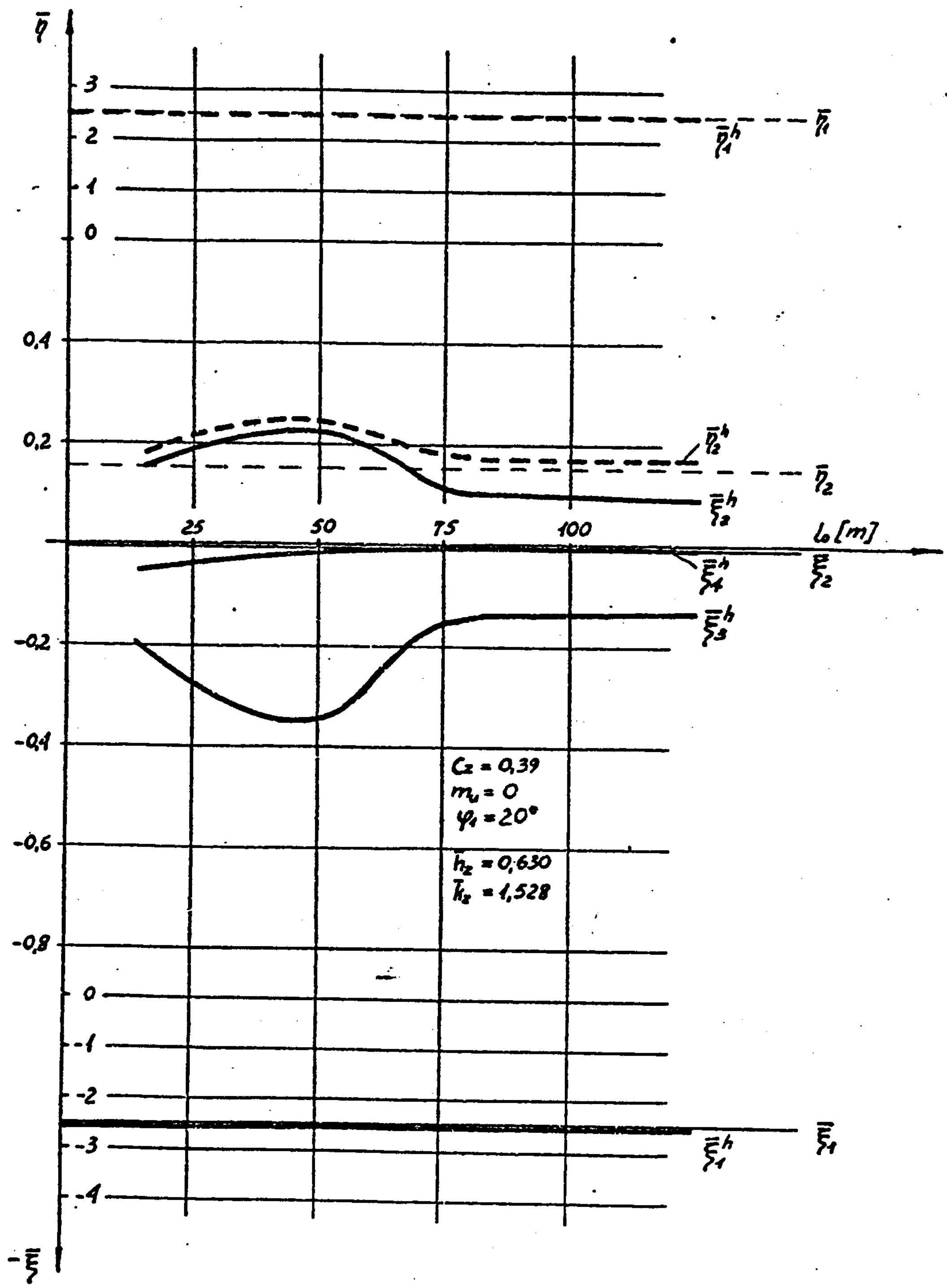

Figure 15. Variation in Damping and Frequency Coefficients of a Glider with Length of Towing Cable for a Towing Angle of $\phi_1 = 20°$

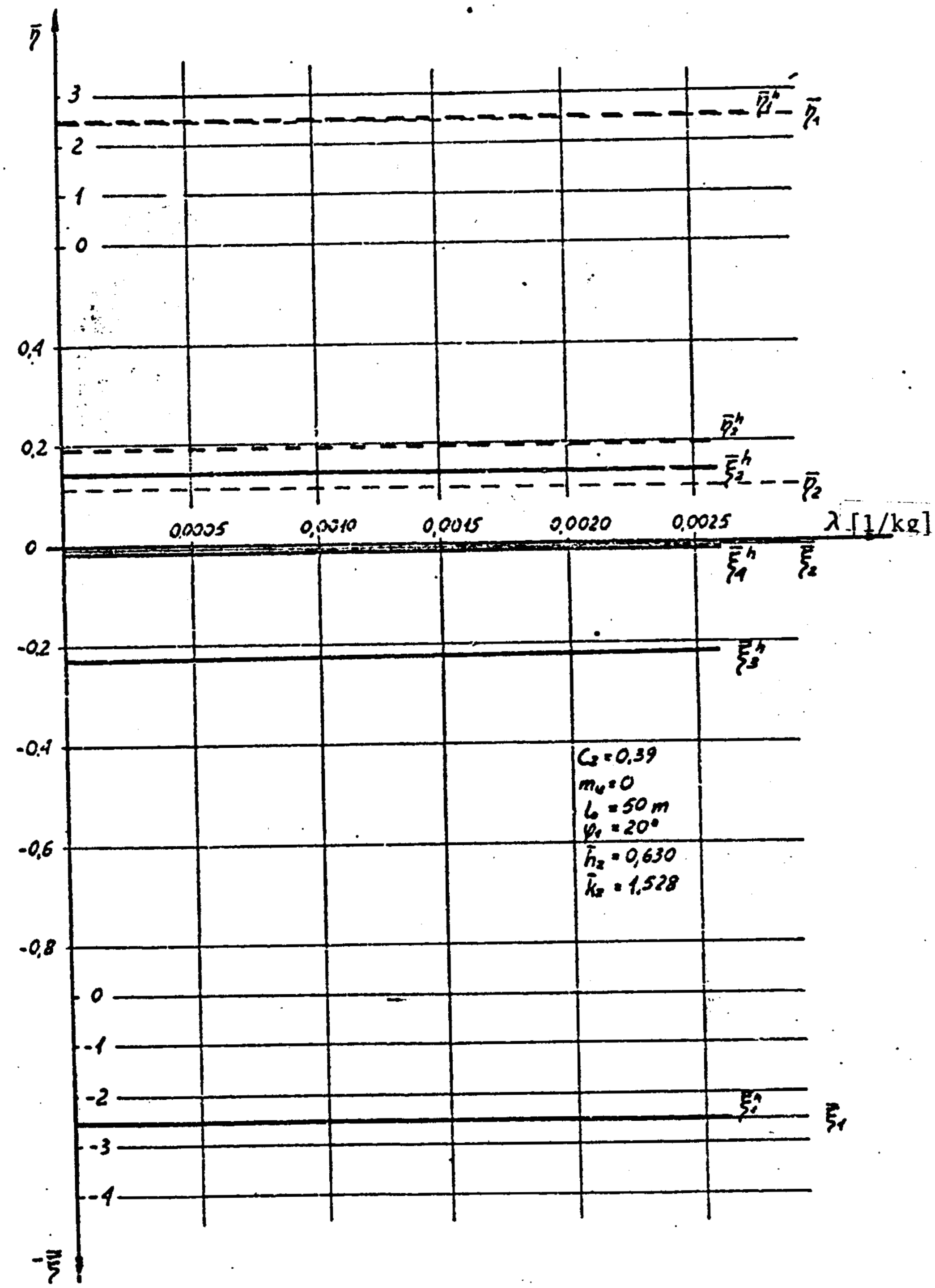

Figure 16. Variation in Damping and Frequency Coefficients of a Glider with Coefficient of Extensibility of a Type C Cable for a Towing Angle of $\phi_1 = 20°$

With a constant towing angle ϕ_1 = 20° the variations in the static margin /89
(Figure 12) and the eigenvalues (Figure 13) have been calculated for various towing speeds, and the eigenvalues for a towing angle of ϕ_1 = -5° (Figure 14).

We see that variation in the towing speed has no effect on the damping of fast oscillations ($\xi_1^h = \bar{\xi}_1$) and a slight effect on variation in frequency ($\bar{\eta}_1^h \approx \bar{\eta}_1$), produces no differences between towed and free flight, and exerts no effect on aperiodic vertical movements ($\bar{\xi}_4^h$ = const) (Figures 13 and 14). On the other hand, increase in the damping of phugoid oscillations $\bar{\xi}_2^h$ occurs with increase in speed, especially for a glider position below the line of flight of the towing aircraft (Figures 13 and 14). The variation in the static margin (Figure 12) is of the same nature as for free flight.

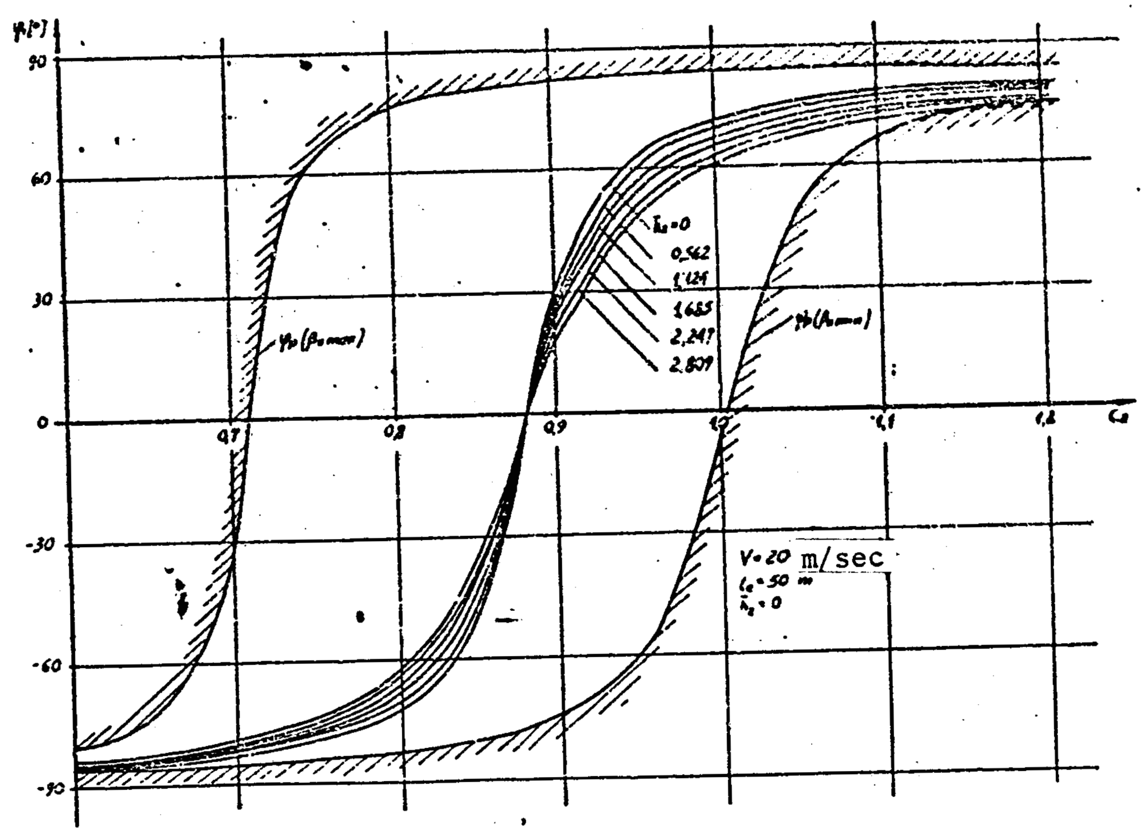

Figure 17. Curves of Glider Equilibrium and Variation in Towing Angle ϕ_1 As a Function of the Lift Coefficient C_z for Various Horizontal Positions of Towing Attachment Relative to Center of Gravity of Glider

9.3. *Effect of length and extensibility of towing cable on glider stability.* Calculations have been made for a glider towed at a speed of V = 30 m/sec at a towing angle of $\phi_1 = 20°$. The towing cable is a type C cable of variable /90 length (Figure 15). Then a type C cable of a length l_0 = 50 m is adopted and the coefficient of extensibility λ is varied for it (Figure 16).

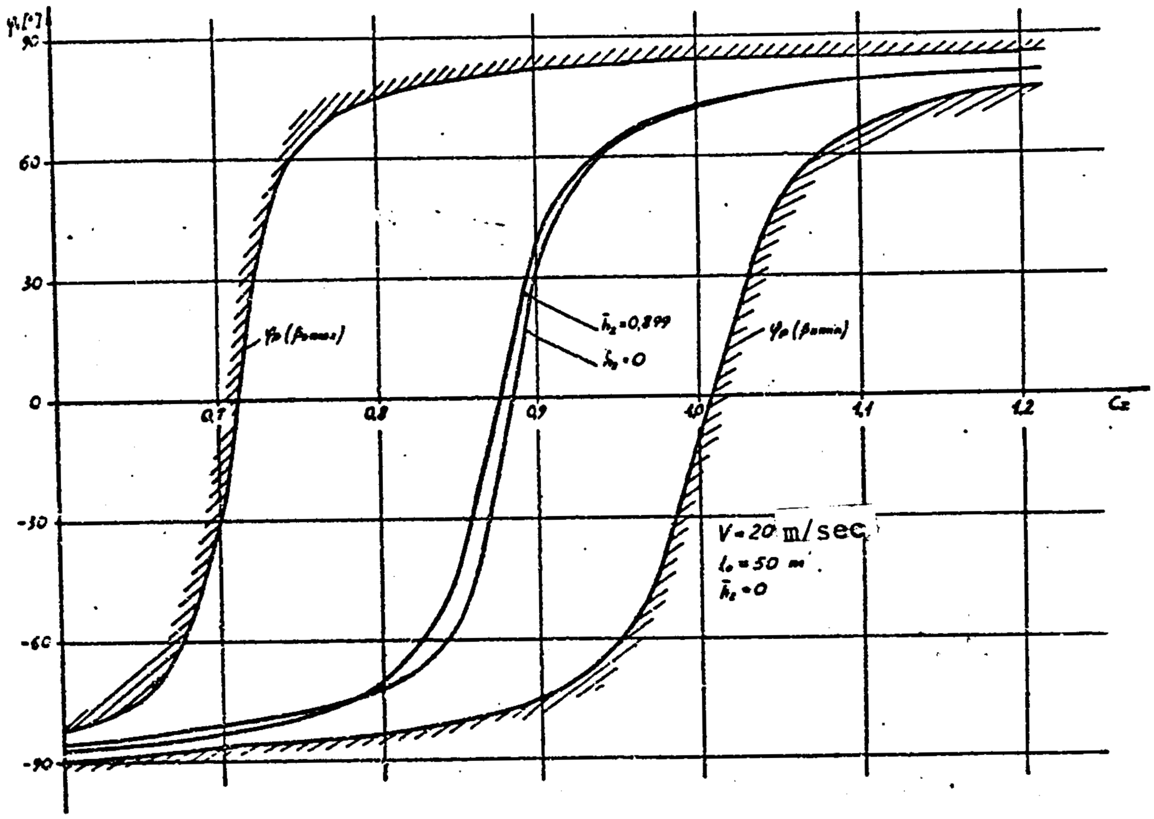

Figure 18. Curves of Glider Equilibrium for Various Vertical Positions of Towing Attachment Relative to Center of Gravity of Glider

Variations in the length of the towing cable and its extensibility cause no changes in the static margin as compared to free flight (if the flight is made at a constant towing angle ϕ_1 = const).

Variation in cable length affects only the phugoid oscillations $(\bar{\xi}_2^h, \bar{\eta}_2^h)$ and aperiodic longitudinal displacements $(\bar{x}_3^h)$, but exerts no effect on rapid oscillations (Figure 15). The extensibility of the cable has no influence on variation in the eigenvalues (Figure 16).

9.4. *Influence of position of towing attachment relative to center of gravity of glider on equilibrium and stability.* The case has been considered of towed flight at a speed of V = 30 m/sec on a type C cable of a length l_0 = 50 m at a constant towing angle ϕ_1 = 20°. The position of the towing cable is varied relative to the center of gravity by displacing it from the center of gravity horizontally forward $\bar{k}_z$ at constant $\bar{h}_z$ = 0 and vertically downward $\bar{h}_z$ at constant $\bar{k}_z$ = 0.

Curves of equilibrium calculated from (6.8) as a function of horizontal displacement of the towing attachment are shown in Figure 17. The influence of vertical displacement of the attachment is shown in Figure 18. We see that displacement of the towing attachment forward causes steeper slope of the equilibrium curves, this in effect permitting greater movement of the elevator to achieve equilibrium. This is an important factor for training gliders. Vertical displacement of the towing attachment has but slight effect on equilibrium.

Displacement of the towing attachment horizontally forward always causes great increase in the *static margin* (Figure 19), while displacement of the attachment vertically downward always reduces the static margin (Figure 20). /195

Variations in the position of the towing attachment exert no effect on rapid oscillations ($\bar{\xi}^h_1$, $\bar{\eta}^h_1$), while they exert a strong influence on phugoid oscillations ($\bar{\xi}^h_2$, $\bar{\eta}^h_2$) and aperiodic displacements ($\bar{\xi}^h_3$ and $\bar{\xi}^h_4$).

Displacement of the attachment vertically forward affects the damping ($\bar{\xi}^h_2 < 0$) of phugoid oscillations and causes increase in the divergence of aperiodic movements ($\bar{\xi}^h_3 > 0$ and $\bar{\xi}^h_4 > 0$). Variation in the position of the attachment vertically downward from the center of gravity causes increase in damping of aperiodic movement and divergence of the phugoid oscillations.

It is to be seen from the foregoing results that the forward position of the towing attachment is more advantageous for reasons of equilibrium and stability of the glider in towed flight.

9.5. *Influence of variation in static margin on dynamic stability of a glider*. The static margin of a glider is affected by the design parameters, the aerodynamic characteristics, and the towing conditions.

Figures 23 and 24 illustrate the variations in the eigenvalues $\bar{\lambda}^h$ in the case of towing at a speed corresponding to the lift coefficient C_z = 0.75 for two glider positions, above (Figure 23) and below (Figure 24) the line of flight of the towing aircraft.

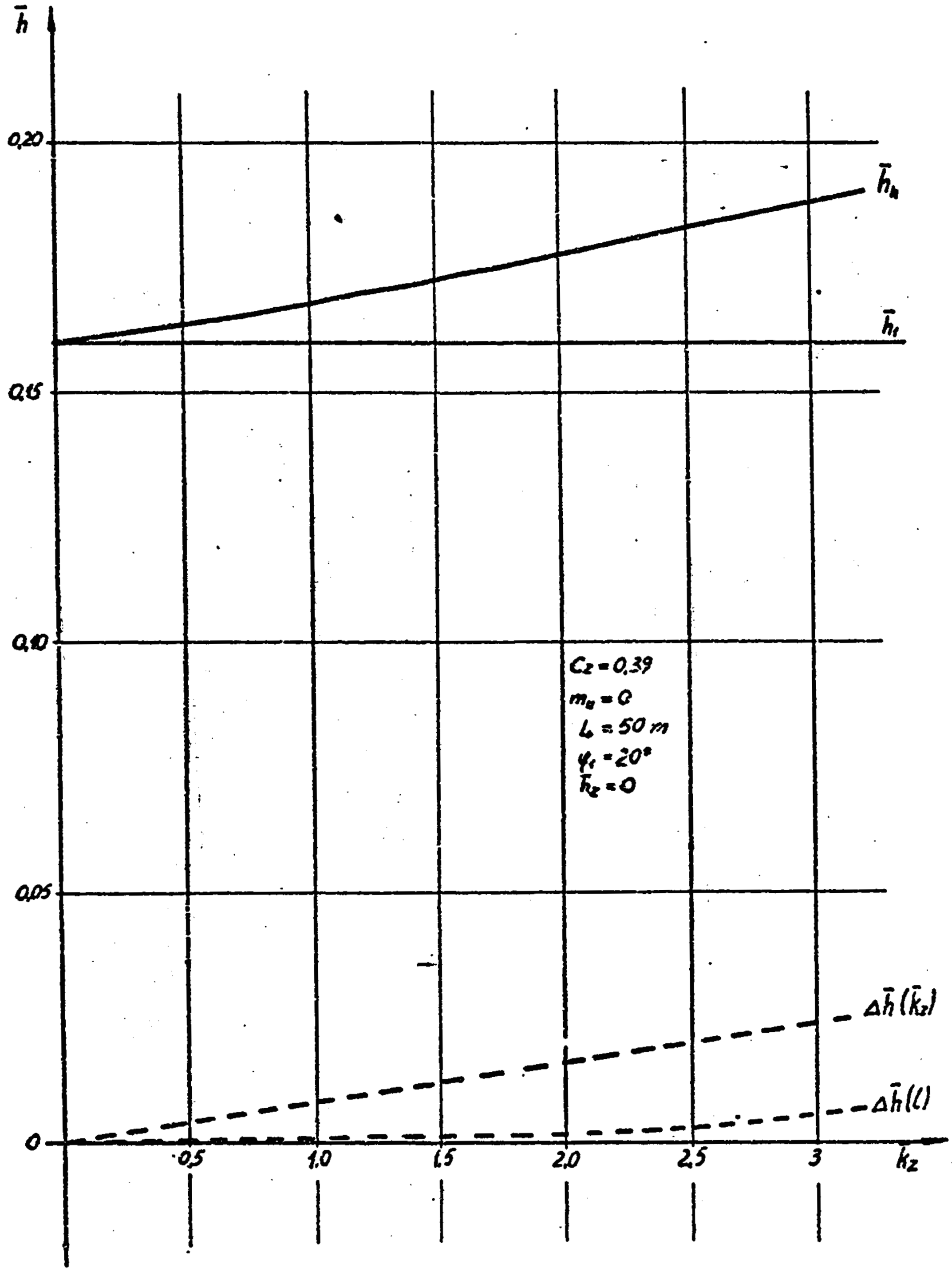

Figure 19. Variation in "Static Margin" with Horizontal Position of Glider Towing Attachment for a Towing Angle $\phi_1 = 20°$

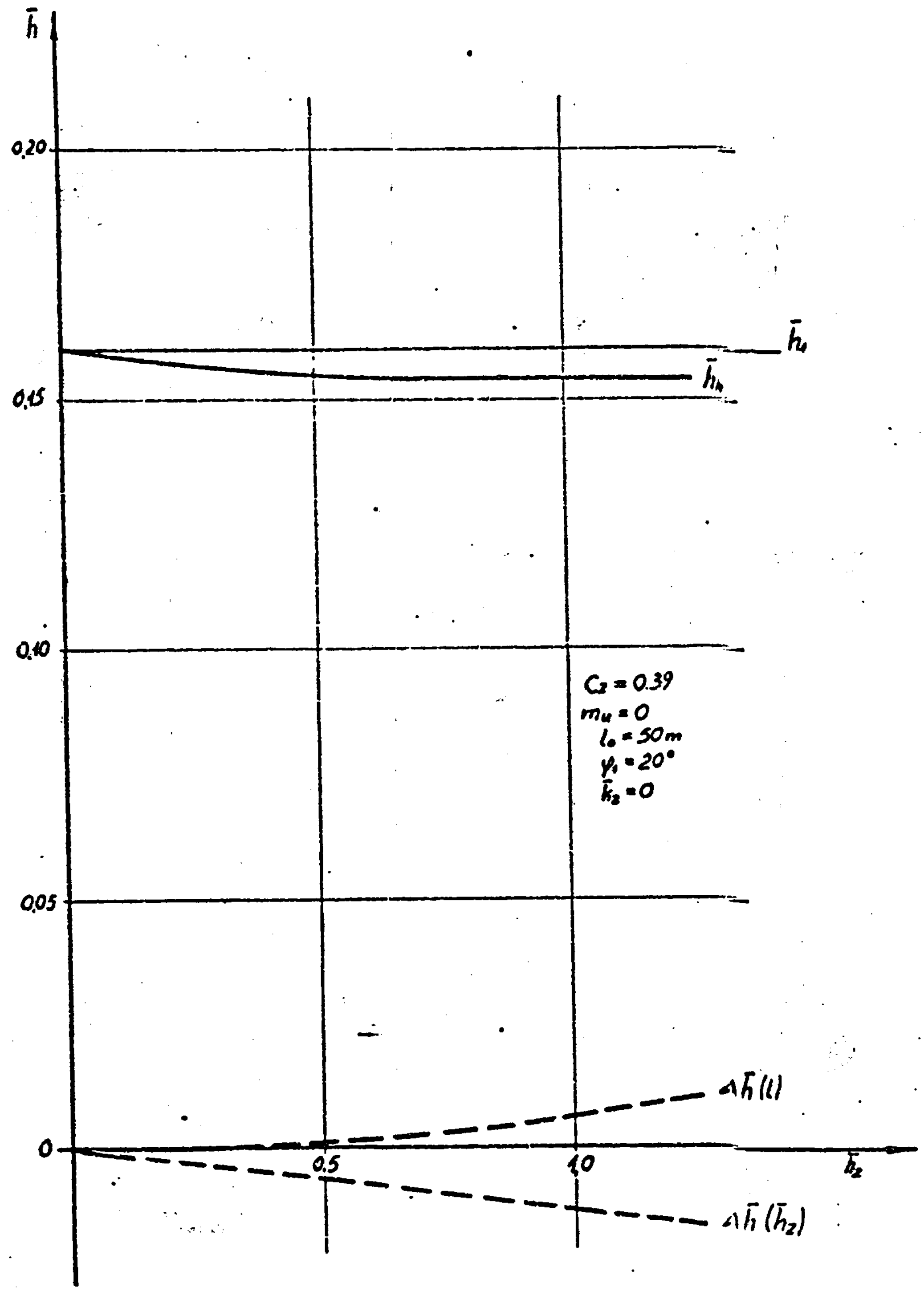

Figure 20. Variation in "Static Margin" with Vertical Position of Glider Towing Attachment for a Towing Angle ϕ_1 = 20°

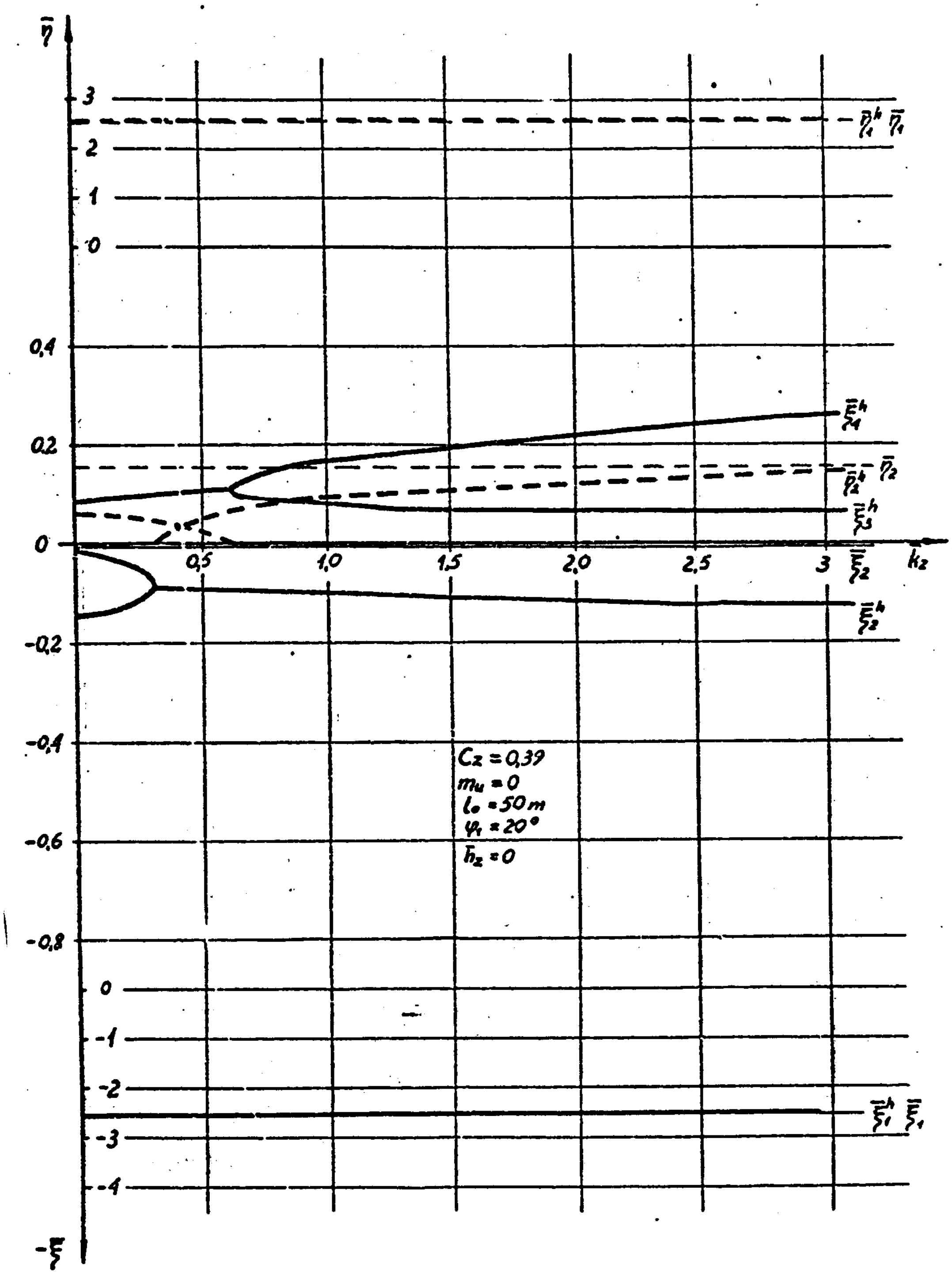

Figure 21. Variation in Glider Damping and Frequency Coefficients with Horizontal Position of Glider Towing Attachment for a Towing Angle $\phi_1 = 20°$

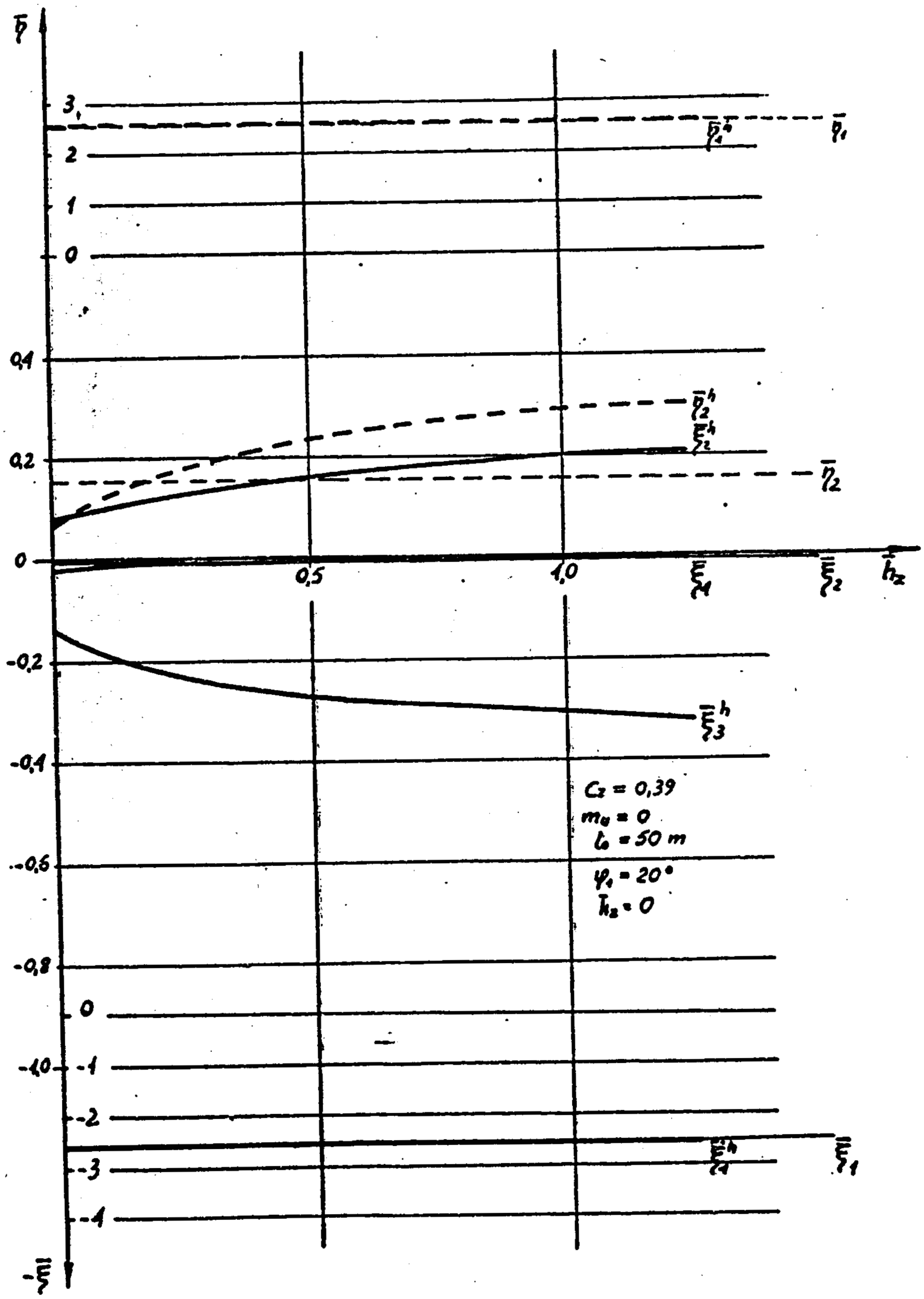

Figure 22. Variation in Glider Damping and Frequency Coefficients with Vertical Position of Glider Towing Attachment for a Towing Angle $\phi_1 = 20°$

Figures 25 and 26 show the $\bar{\lambda}^h$ function for the same glider positions, but with a lift coefficient $C_z = 0.22$, this corresponding to higher speed.

It follows from these diagrams that both the damping and the frequency of rapid oscillations in towed and free flight are identical (Figures 23-26).

Increase in the static margin exerts a stabilizing effect, since there is a decrease in positive values $\bar{\xi}^h$.

An important factor affecting stability is the towing speed (flight at smaller values C_z) and the position of the glider relative to the towing aircraft.

Comparing the results of Figure 26 with the results of Figure 23-25 we see that the most advantageous conditions of towed flight are higher towing speed (in the glider case considered $V_h > 1.4\ V_{min}$) and a position of the glider below the line of flight of the towing aircraft.

9.6. *Conclusions.* The stability of a glider in towed flight is influenced by the towing parameters and by the structural and aerodynamic design of the glider. The considerations of the foregoing sections point to the following conclusions.

a. *Design.* The glider towing attachment should be situated as far as possible forward from the center of gravity (and insofar as possible above the center of gravity of the glider). This causes increase in the static margin and damping of phugoid oscillations, and prevents abrupt variations in the vertical position of the glider with small movements of the control stick.

The static margin insuring proper piloting of a glider in free flight is sufficient for towed flight.

b. *Conclusions deriving from variations in towing parameters.* Because of the possibility of earlier *stall* of a glider in towed flight, the minimum towing speed must be determined for the particular glider, $V_{h\ min} > V_{min}$.

Higher towing speeds insure increase in glider stability.

A glider position below the line of flight of the towing aircraft is more advantageous, since it causes damping of the phugoid oscillations. /100

Instability of the phugoid oscillations or slight divergence of the aperiodic movements may occur in towed flight, but this takes place very slowly and the pilot can always react with the controls to increase stability.

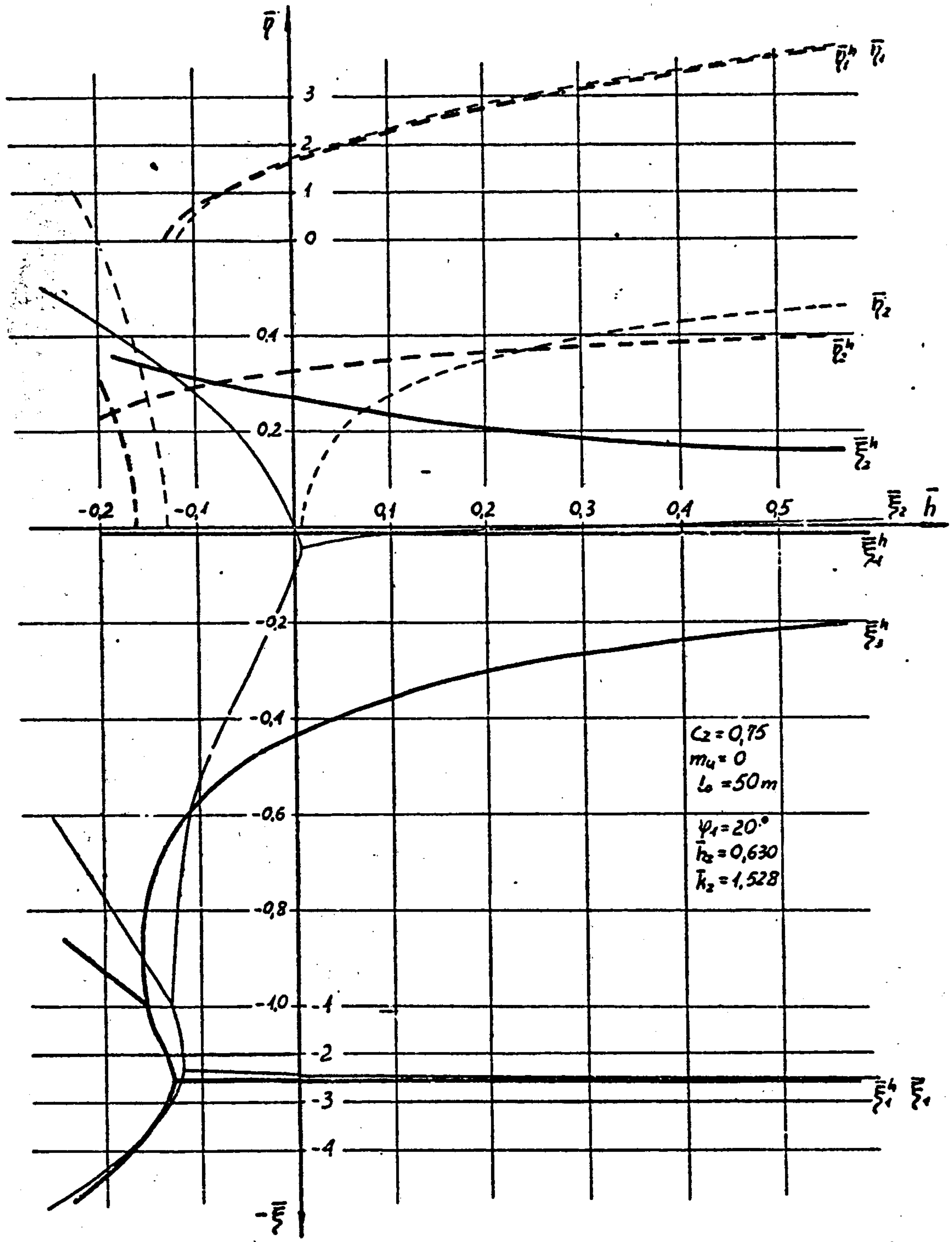

Figure 23. Variation in Coefficients of Damping and Frequencyof a Glider with "Static Margin" for a Towing Angle ϕ_1 = 20° and C_z = 0.75

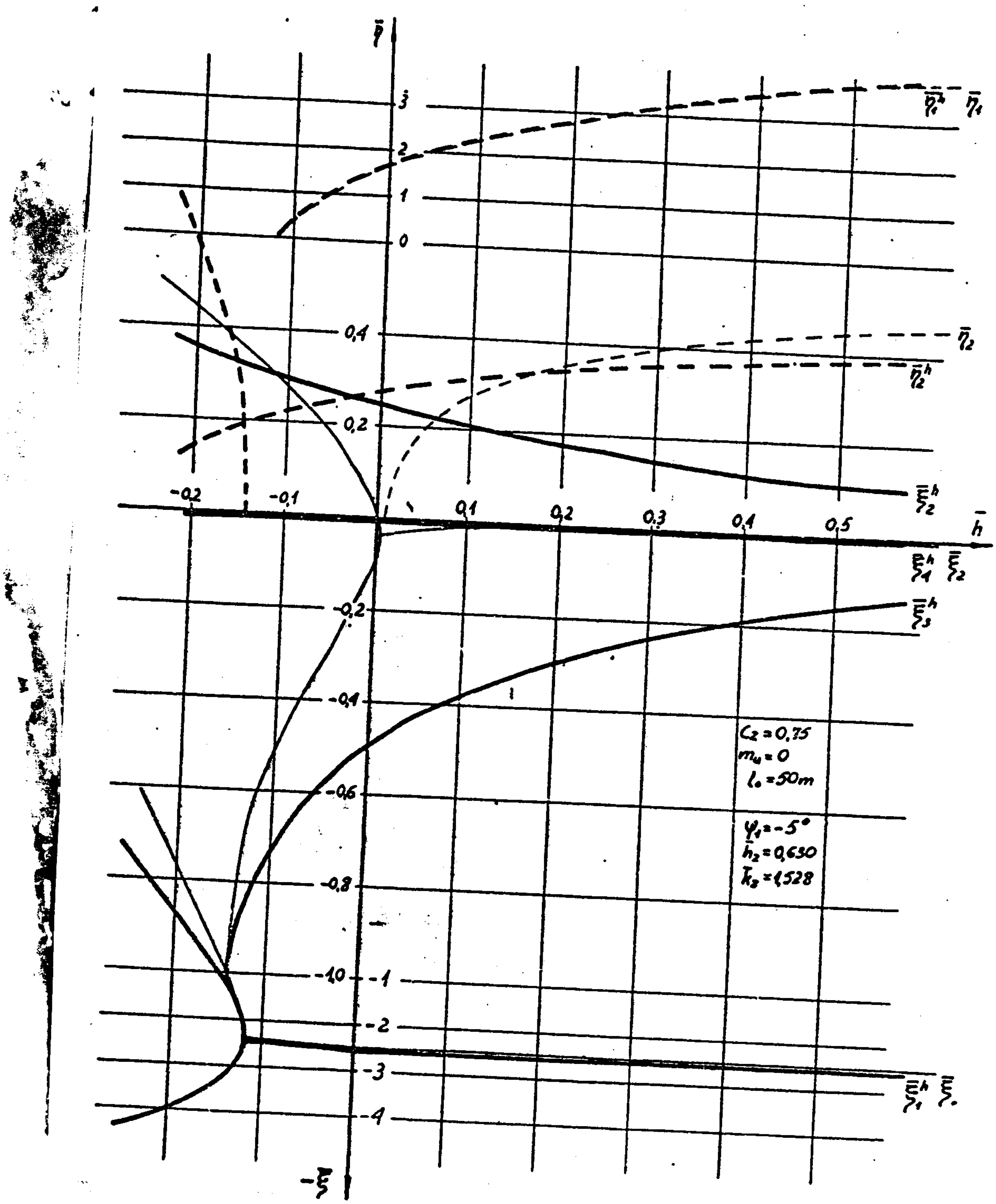

Figure 24. Variation in Coefficients of Damping and Frequency of a Glider with "Static Margin" for a Towing Angle $\phi_1 = -5°$ and $C_z = 0.75$

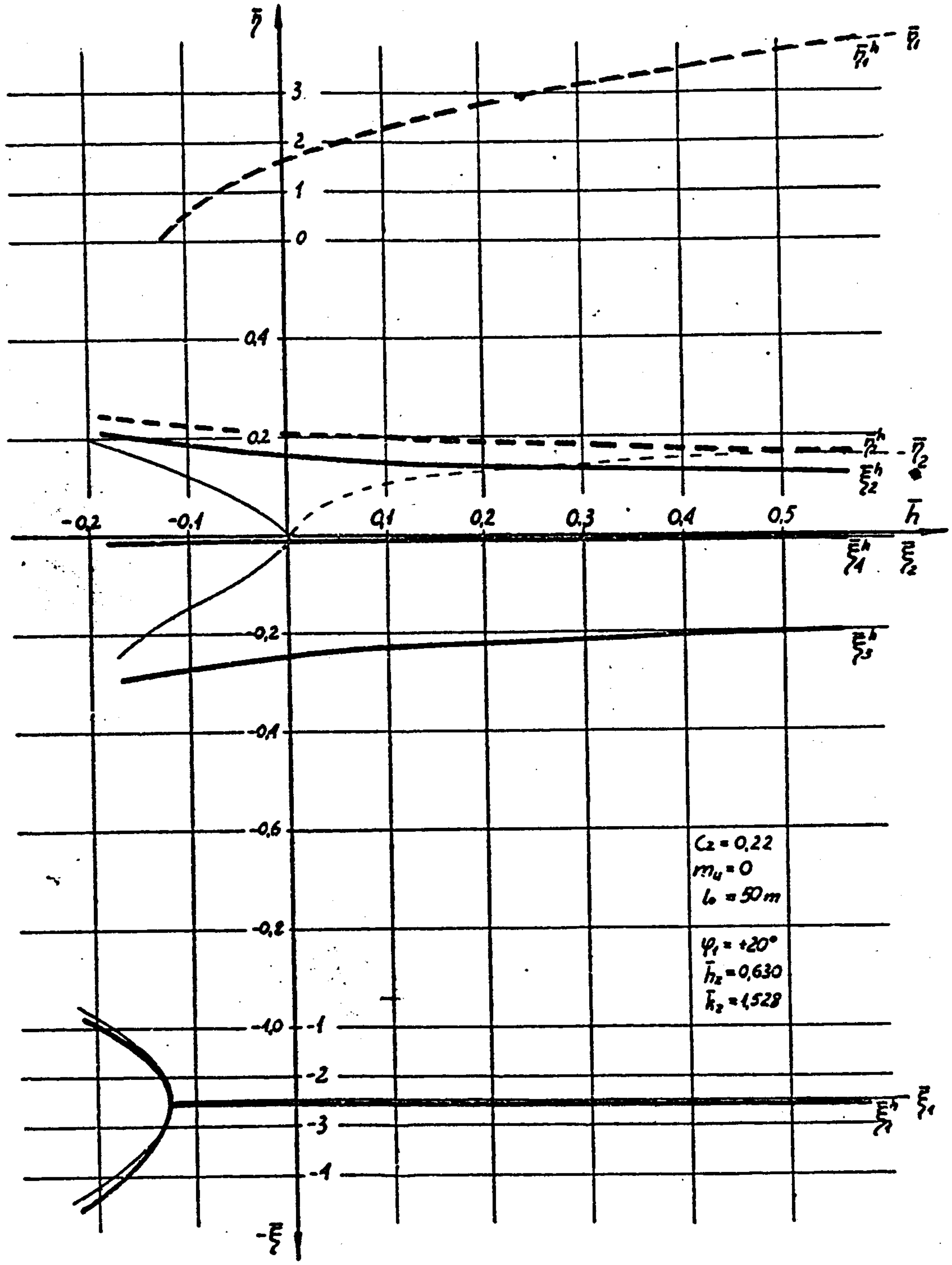

Figure 25. Variation in Coefficients of Damping and Frequency of a Glider with "Static Margin" for a Towing Angle ϕ_1 = 20° and C_z = 0.22

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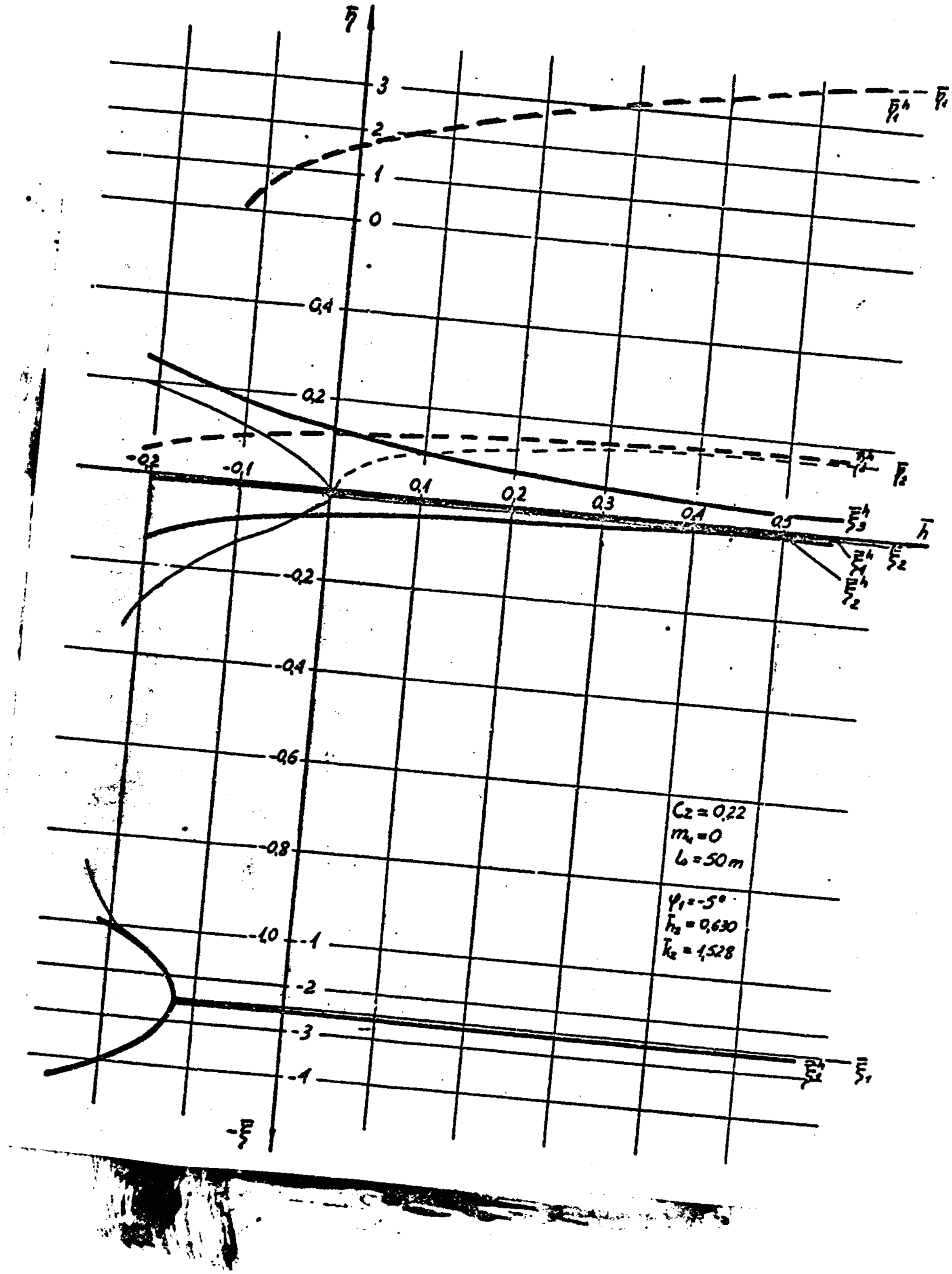

Figure 26. Variation in Coefficients of Damping and Frequency of a Glider with "Static Margin" for a Towing Angle $\phi_1 = -5°$ and $C_z = 0.22$

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